

# An Improved Approach to Kalman Bucy Filter using the Identification Algorithm

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*The Kalman Bucy filter is a well-known observer to estimate the state vector from the incomplete state measurements. However, when the time delay is taken into account, the filter can become ineffective. In this paper, the identification algorithm presented in a previous paper (Anh 2000) is used to improve the Kalman Bucy filter in the presence of time delay. The differential equation of the observer error vector is expanded by modal eigenfunction technique. Using the identification algorithm, the external excitation acting on some first modes is identified with a time delay and with a small error depending on the sensor locations. Then the identified excitation is eliminated from the observer equation. A numerical calculation is applied to an eight story building subjected to base acceleration and controlled by active mass damper system. In the presence of time delay, the comparison between performance indexes shows the effectiveness of improved Kalman Bucy filter to the classical Kalman Bucy filter*

## 1 Introduction

In recent years, much progress and new concepts have been achieved in the development of structural control in reducing the response during excessive vibrations due to environmental disturbances (Spencer et al 1997, Housner et al 1997). The control of structure motions can be done by various means. Among them feedback active control is one of the promise aspects which uses the control counterforces produced by actuators to balance or reduce the energy of the environmental loading. Most of civil engineering structural control studies have primarily used state feedback control methods. The practical application of these methods to large structures, such as high buildings and cable-stayed bridges, leads to some specific problems. In fact, large structures require high dimensional models to capture their dynamics. Thus, the use of full state feedback active control may require a large number of sensors and actuators that may be impossibly realized in practice. It is often necessary to replace full state vector by state estimator determined from incomplete state measurements. The state estimator can be designed as a Luenberger observer or as a Kalman Bucy filter (optimal observer) (Luenberger 1966, Kalman and Bucy 1961, Kwakernaak et al 1972). These observers use a linear feedback of the difference between the measurement output and computed output to reduce the estimator error. This, generally, be achieved by locating the observer poles quite deep in the left half complex plane, which implies a large observer gain matrix. However, a large observer gain matrix makes the observer sensitive to sensor noise, calculation error or time delay. A number of research activities have considered time delay in observer design, (for example Watanabe et al 1985, Mahmoud et al 1999, Pila et al 1999, Wang et al 2001,2002, Fridman et al 2001,2003, Leyva-Ramos et al 1995). The basic tools used mainly in these works were the robust ( $H_\infty$ ) filtering theory.

In fact, we presented a method called identification algorithm (Anh 2000), which identifies the external excitation from the structural response measured. Thus, the aim of this paper is to improve the Kalman Bucy filter or optimal observer by the identification algorithm. Using the identification algorithm, the reduction of the estimator error can be achieved without requirement of large observer gain matrix. The numerical calculation is applied to an eight-story building controlled by an active mass system. In the example, by considering the time delay of measurement output, the performance of improved Kalman filter is better than that of classical Kalman filter because the first one does not require large observer gain matrix.

## 2 Control using Kalman-Bucy Filter

Considering a linear control problem of the state space form:

$$\dot{z}(t) = Az(t) + Bu(t) + Hf(t) \quad (1)$$

$$y(t) = Cz(t) \quad (2)$$

where  $z(t)$  is the  $n$ -dimensional state vector,  $f(t)$  is the  $r$ -dimensional external force vector,  $u(t)$  is the  $m$ -dimensional control force vector,  $A$  is an  $n \times n$  system matrix,  $B$  and  $H$  are  $n \times m$  and  $n \times r$  location matrices specifying the locations of controllers and external excitations in the state-space, respectively. The  $p$ -dimensional measurement vector  $y(t)$  is defined by the  $p \times n$  measurement matrix  $C$ . It is noted that, in most cases,  $p < n$ , implies that the complete state  $z(t)$  can not directly calculate from the incomplete measurement  $y(t)$ . Using state feedback control methods, the control force  $u(t)$  is chosen as

$$u(t) = Gz(t) \quad (3)$$

where  $G$  is the control gain matrix, which depends on the control algorithm selected. However, because it is not practical to measure the full state vector, the control law (3) should be replaced by

$$u(t) = G\hat{z}(t) \quad (4)$$

where  $\hat{z}(t)$  is the state estimator designed as an observer, which is given by

$$\dot{\hat{z}}(t) = A\hat{z}(t) + Bu(t) + G_e(y(t) - C\hat{z}(t)). \quad (5)$$

In the above, the estimator has an internal model of the system as indicated by the first two terms, which are prediction terms. The third term corrects the model by a linear feedback of the difference between the measurement output  $y(t)$  and the computed output  $C\hat{z}(t)$ . This is the correction term. It has been seen that a large observer gain matrix  $G_e$  makes the observer sensitive to sensor noise, calculation error or time delay. The Kalman Bucy filter (first solved by Kalman and Bucy 1961) is an observer, in which the observer gain matrix  $G_e$  is chosen optimal in some sense. A relatively simple formulation of observer gain matrix described below is taken from Kwakernaak et al 1972

$$G_e = P_e C^T V^{-1} \quad (6)$$

where  $P_e$  satisfies the algebraic matrix Riccati equation

$$AP_e + P_e A^T + HFH^T - P_e C^T V^{-1} C P_e = 0 \quad (7)$$

in which,  $F$  and  $V$ , respectively, are appropriate  $r \times r$  and  $p \times p$  weighting matrices. In the Kalman Bucy filter derivation,  $F$  and  $V$ , respectively, are considered as the intensities of the state excitation white noise and sensor white noise. When no sensor noise presents as seen in (2), the weighting matrix  $V$  approaches zero and the observer gain matrix  $G_e$  in equation (6) becomes infinite. However, a too large gain matrix can not only amplify the time delay in processing measured information but may also cause instability in the system. This problem is discussed more detailed in section 4. The block diagram of the control problem using Kalman filter is shown in Figure 1.

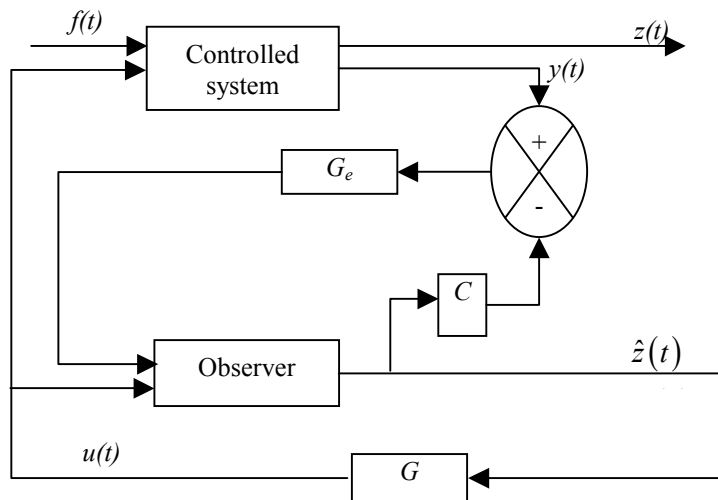


Figure 1. Block diagram of a control system using Kalman Bucy filter

Instead of making a large gain matrix, in this paper, we will improve the Kalman Bucy filter by a feedforward term, which depends on the external excitation. The details are presented below.

### 3 Control using Kalman Filter with the Addition of a Feedforward Term

We propose to add a new variable  $u_f(t)$  to the observer (5). We have

$$\dot{\hat{z}}(t) = A\hat{z}(t) + Bu(t) + G_e(y(t) - C\hat{z}(t)) + u_f(t) \quad (8)$$

denoting the estimation error as

$$e(t) = z(t) - \hat{z}(t). \quad (9)$$

Subtracting (8) from (1), considering (2) and (9), we obtain

$$\dot{e}(t) = (A - G_e C)e(t) + Hf(t) - u_f(t). \quad (10)$$

We also introduce the following notation

$$y_e(t) = y(t) - C\hat{z}(t) = Ce(t). \quad (11)$$

Two equations (10) and (11) make a control system, in which the controlled variable is  $e(t)$ , the measured variable is  $y_e(t)$ , the excitation is  $Hf(t)$  and the input variable is  $u_f(t)$ . The control problem is to find the input variable  $u_f(t)$  necessary to reduce the norm of estimator error  $\|e(t)\|$ . The idea is that, the input variable  $u_f(t)$  should be chosen to eliminate the excitation  $Hf(t)$ . Because the input variable depends only on external excitation,  $u_f(t)$  is called the feedforward term. However, when only the measurement vector in equation (11) can be measured, the excitation can not be known all. In this paper, by using modal superposition method, some most important excitations is identified and eliminated. The detail is presented as following. Let matrix  $A - G_e C$  has distinct eigenvalues  $\lambda_j (j=1,..n)$  and corresponding eigenvectors  $\eta_j$ . Assuming that the eigenvalues  $\lambda_j (j=1,..n)$  are ordered such as

$$|\lambda_1| \leq |\lambda_2| \leq \dots \leq |\lambda_n|.$$

Defining the  $n \times p$  matrix  $\Phi_c$ , the  $n \times (n-p)$  matrix  $\Phi_r$ , the  $p \times n$  matrix  $\Psi_c$ , the  $(n-p) \times n$  matrix  $\Psi_r$ , the  $p \times p$  diagonal matrix  $\Lambda_c$  and the  $(n-p) \times (n-p)$  diagonal matrix  $\Lambda_r$  by

$$\begin{aligned} \Phi_c &= [\eta_1 \ \eta_2 \ \dots \ \eta_p]; \Phi_r = [\eta_{p+1} \ \eta_{p+2} \ \dots \ \eta_n]; [\Phi_c \ \Phi_r]^{-1} = \begin{bmatrix} \Psi_c \\ \Psi_r \end{bmatrix}; \\ \Lambda_c &= \text{diag}[\lambda_1 \ \lambda_2 \ \dots \ \lambda_p]; \Lambda_r = \text{diag}[\lambda_{p+1} \ \lambda_{p+2} \ \dots \ \lambda_n]. \end{aligned} \quad (12)$$

Then

$$A - G_e C = [\Phi_c \ \Phi_r] \begin{bmatrix} \Lambda_c & 0 \\ 0 & \Lambda_r \end{bmatrix} \begin{bmatrix} \Psi_c \\ \Psi_r \end{bmatrix}$$

Applying the modal transformation

$$\begin{bmatrix} e_c(t) \\ e_r(t) \end{bmatrix} = \begin{bmatrix} \Psi_c \\ \Psi_r \end{bmatrix} e(t)$$

the state equation (10) is decoupled.

$$\dot{e}_c(t) = \Lambda_c e_c(t) - \Psi_c u_f(t) + f_c(t) \quad (13)$$

$$\dot{e}_r(t) = \Lambda_r e_r(t) - \Psi_r u_f(t) + f_r(t) \quad (14)$$

in which,  $f_c(t)$  and  $f_r(t)$ , respectively, are the excitation in the modal space

$$f_c(t) = \Psi_c H f(t); f_r(t) = \Psi_r H f(t).$$

The difference measurement vector  $y_e(t)$  is also rewritten in modal space

$$y_e(t) = C_c e_c(t) + C_r e_r(t) \quad (15)$$

where

$$C_c = C \Phi_c; C_r = C \Phi_r.$$

As one knows, the vibrational modes corresponding to large eigenvalues often contribute insignificantly to the response (Soong 1989), so attention needs to be paid only to a few vibrational modes. Thus, the excitation term  $f_c(t)$  need to be identified. Combining (13) and (15), we have

$$f_c(t) = C_c^{-1} \dot{y}_e(t) - \Lambda_c C_c^{-1} y_e(t) + \Psi_c u_f(t) - C_c^{-1} C_r \dot{e}_r(t) + \Lambda_c C_c^{-1} C_r e_r(t). \quad (16)$$

We assume that the feedforward term  $u_f(t)$  can be known, which will be discussed later, the difference measurement vector  $y_e(t)$  in (15) is known and its first derivatives can be calculated. Therefore, the first three terms in equation (16) can be known while the last two terms can not be calculated. We denote the excitation estimator  $\hat{f}_c(t)$  as

$$\hat{f}_c(t) = C_c^{-1} \dot{y}_e(t) - \Lambda_c C_c^{-1} y_e(t) + \Psi_c u_f(t). \quad (17)$$

The error  $E(t)$  of the identification process is considered as the difference between the exact excitation and the estimated excitation

$$E(t) = f_c(t) - \hat{f}_c(t) = \Lambda_c C_c^{-1} C_r e_r(t) - C_c^{-1} C_r \dot{e}_r(t). \quad (18)$$

To attenuate this error term, the following conditions should be satisfied:

- The sensors should be located to obtain a large norm of  $C_c$  in comparison with the norm of  $C_r$ .
- The insignificant vibrational modes  $e_r(t)$  should be small enough

The problem remains is to determine the feedforward term  $u_f(t)$ . From equation (13), it is seen obviously that the feedforward term should be chosen to eliminate the identified excitation

$$-\Psi_c u_f(t) + f_c(t) = 0 \text{ or } \Psi_c u_f(t) = f_c(t). \quad (19)$$

However, because we can not know the exact excitation  $f_c(t)$ , but only the estimate  $\hat{f}_c(t)$ , the equation (19) should be modified as

$$\Psi_c u_f(t) = \hat{f}_c(t). \quad (20)$$

Besides, to ensure the stability of the controlled system, the feedforward term  $u_f(t)$  must be chosen to make the error  $E(t)$  in (18) not depend on  $u_f(t)$ . This means that the feedforward term  $\Psi_r u_f(t)$  acting on the insignificant vibrational mode  $e_r$  must be set to zero for the entire time duration

$$\Psi_r u_f(t) = 0. \quad (21)$$

Using (12), (20) and (21), we determine  $u_f(t)$  by transformation from modal space to state space

$$u_f(t) = [\Phi_c \quad \Phi_r] \begin{bmatrix} \Psi_c \\ \Psi_r \end{bmatrix} u_f(t) = \Phi_c \Psi_c u_f(t) + \Phi_r \Psi_r u_f(t) = \Phi_c \hat{f}_c(t). \quad (22)$$

However, the difficulty arises because of the dependence between  $u_f(t)$  and  $\hat{f}_c(t)$  from (17) and (22). The idea involved in the expression (22) may be used in a modified way, in which the history of the external excitation can be identified with a time delay by the process presented in (Anh 2000). The modification of equation (22) is

$$u_f(t) = \Phi_c \hat{f}_c(t - \Delta) \quad (23)$$



$$u_f(t) = \Phi_c C_c^{-1} (\dot{y}(t - \Delta - \tau) - C\dot{z}(t - \Delta)) - \Phi_c \Lambda_c C_c^{-1} (y(t - \Delta - \tau) - C\hat{z}(t - \Delta)) + u_f(t - \Delta). \quad (27)$$

The error caused by time delay can be amplified through the term  $\Phi_c \Lambda_c C_c^{-1} (y(t - \Delta - \tau) - C\hat{z}(t - \Delta))$ . However, as indicated above, the improved Kalman filter does not require too large gain matrix.

## 5 Numerical Calculation

The example given below is taken from (Yang 1982). An eight-storey structure in which every storey unit is identically constructed is considered. The characteristics of the building are the same for each story: floor mass  $m$ , elastic stiffness  $k$  and internal damping coefficient  $c$ . Assuming that the structure is subject to the earthquake ground acceleration, whose history is taken from the N-S component recorded at Hachinohe City during the Tokachioki earthquake of May 16, 1968. The absolute peak acceleration of the earthquake record is  $2.25\text{m/s}^2$ . The control is accomplished through an active mass damper system installed at the top of the structure as shown in Figure 3. An active mass damper (AMD) is a system, in which an auxiliary mass  $m_d$  is connected to the main structure through a spring  $k_d$ , a damping device  $c_d$  and a hydraulic actuator producing an active force  $u$ . Without the active force, the mass damper is passive and is called tuned mass damper (TMD). Passive TMD system was widely used for motion control of tall buildings (Soong et al 1997). Therefore, AMD system is also the most popular mechanism in active structural control (Spencer et al 1997). The optimum values of spring and damping device are available in literature. They are in general tuned to the first fundamental frequency of the structure.

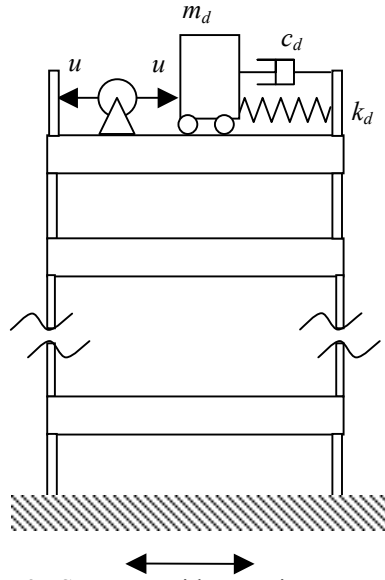


Figure 3. Structure with an active mass damper

It is not difficult to derive the structural motion equation having the form

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = L_u u(t) + L_f \ddot{x}_g(t) \quad (28)$$

where the mass, damping and stiffness matrices have form

$$M = m \begin{bmatrix} 8 + \mu & 7 + \mu & 6 + \mu & \dots & 1 + \mu & \mu \\ 7 + \mu & 7 + \mu & 6 + \mu & \dots & 1 + \mu & \mu \\ 6 + \mu & 6 + \mu & 6 + \mu & \dots & 1 + \mu & \mu \\ \vdots & \vdots & \vdots & \ddots & 1 + \mu & \mu \\ 1 + \mu & 1 + \mu & 1 + \mu & 1 + \mu & 1 + \mu & \mu \\ \mu & \mu & \mu & \mu & \mu & \mu \end{bmatrix};$$

$$D = \text{diag}(c, c, \dots, c, c_d); K = \text{diag}(k, k, \dots, k, k_d)$$

in which  $\mu = m_d/m$  is the mass ratio,  $\ddot{x}_g$  denotes the base acceleration. The displacement vector  $x$ , the location matrices  $L_u$  and  $L_f$  have the form

$$x = [x_1, x_2 - x_1, x_3 - x_2, \dots, x_8 - x_7, x_d - x_8]^T$$

$$L_u = [0 \ 0 \ 0 \ \dots \ 1]^T$$

$$L_f = -[8 + \mu, \ 7 + \mu, \ 6 + \mu, \ \dots \ 1 + \mu, \ \mu]^T m$$

where  $x_i$  ( $i=1,..8$ ) and  $x_d$ , respectively, are the relative displacement of the  $i$ th floor and the auxiliary mass with respect to the foundation. The motion equation (28) then is represented in the state-space form (1)

$$\dot{z}(t) = Az(t) + Bu(t) + Hf(t)$$

where

$$z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}; A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}; B = \begin{bmatrix} 0 \\ -M^{-1}L_u \end{bmatrix}; H = \begin{bmatrix} 0 \\ -M^{-1}L_f \end{bmatrix}; f(t) = \ddot{x}_g(t).$$

The control gain matrix  $G$  in (4) is obtained from the well-known linear quadratic regulator (LQR) control (Kwakernaak et al 1972, Soong 1989). The LQR control algorithm is derived from the minimization of a performance index  $J$ , defined as

$$J = \frac{1}{2} \int_0^{t_f} [z^T(t)Qz(t) + u^T(t)Ru(t)]dt. \quad (29)$$

In the above, the time interval  $[0, t_f]$  is defined to be longer than that of the external excitation,  $Q$  and  $R$  are weighting matrices, whose magnitudes depend on the relative importance attached to the state variables and to the control forces in the minimization procedure. The control gain matrix  $G$  is given by

$$G = -R^{-1}B^T P$$

where  $P$  is the Riccati matrix satisfying the Riccati equation

$$PA - PBR^{-1}B^T P + A^T P + Q = 0.$$

Assuming that, there are two sensors, the first one measures the relative displacement of the top floor with respect to the 7th floor and the second one measures the relative displacement of the auxiliary mass with respect to the top floor. The measurement vector  $y(t)$  contains 2 components. All elements of the measurement matrix  $C$  are zero except  $C_{1,8}$  and  $C_{2,9}$ . Let the parameters take values as (Yang 1982):  $m = 345.6$  metric tons, elastic stiffness  $k = 3.404 \times 10^5$  kN/m, internal damping coefficient  $c = 2937$  metric tons/sec. The first natural frequency of this building is 0.92Hz. For the active mass damper,  $m_d = 29.3$  tons,  $c_d = 25.0$  tons/sec and  $k_d = 957.2$  kN/m. Thus, the damper frequency is tuned to 98% of the first natural frequency of the structure and the damping ratio of the damper is 7.3%. Assuming that the controlled variable is the top floor displacement  $x_8$ . Thus, the performance index  $J$  denoted in (29) is chosen as

$$J = \frac{1}{2} \int_0^{t_f} [x_8^T(t)x_8(t) + u^T(t)Ru(t)]dt.$$

Because there is only one control force, the weighting matrix  $R$  in this case is a scalar and is assigned a value of  $10^{-7}$ . In order to calculate the observer gain matrix  $G_e$  in (6), we need to define weighting matrices  $F$  and  $V$ . The scalar  $F$  and the  $2 \times 2$  matrix  $V$ , respectively, are assigned the values of 1 and  $\rho I_2$ , where  $I_2$  denotes the 2-dimensional identity matrix and  $\rho$  is a positive scalar that is varied. It is intuitively clear that decreasing  $\rho$  improves the speed of state reconstruction but increases the observer gain matrix as shown from (6). As discussed in section 4, a large observer gain matrix can amplify the time delay in processing measured information. To emphasize the deficiency of large gain matrix, the time delay  $\tau$  is taken into account. In case of classical Kalman filter, the equation (26) with  $u_f(t)=0$  is used. In case of improved Kalman filter, the equations (26) and (27) are used. The time delay  $\tau$  is taken from some values: 0 sec (no delay), 0.005 sec, 0.01 sec, 0.015 sec and 0.02 sec. The chosen values, respectively, approximate to 0%, 0.5%, 1%, 1.5% and 2% of the structure's fundamental period. Besides, from equation (25), we also have another time delay  $\Delta$ , which is fixed a value of 0.005s in this example. It should be noted that, the time delay  $\tau$  and  $\Delta$ , respectively, are consumed in processing measured information and performing on-line computation. The total calculation time is 30s. From Figure 1 to Figure 5, the values of performance index  $J$  are shown as the functions of the parameter  $\rho$ . In each figure, 3 cases are considered:

- The case of passive TMD (uncontrolled)
- The case of output feedback control law using the Kalman Bucy filter (discussed in section 2)
- The case of output feedback control law using the Kalman Bucy filter with the addition of feedforward term (discussed in section 3).

The results in Figure 4 are obtained in the absence of time delay ( $\tau=0$ ), in which the system is always stable. From Figure 5 to Figure 8, the time delay  $\tau$  is taken into account with the values 0.005 sec, 0.01sec, 0.015sec and 0.02sec, respectively.

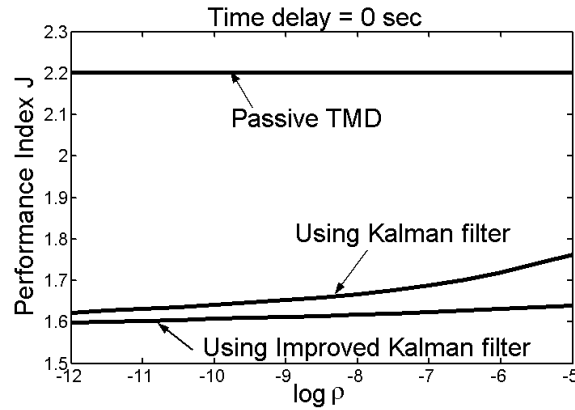


Figure 4. Results in case  $\tau=0$  sec

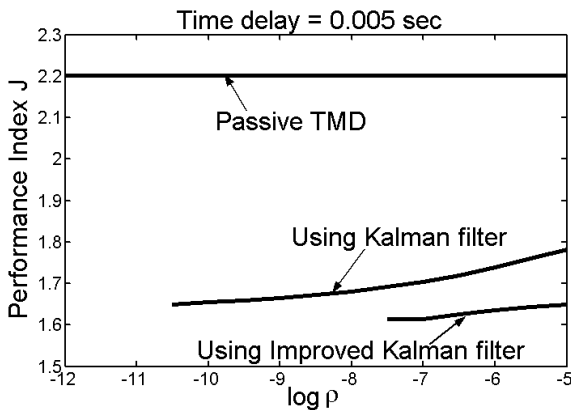


Figure 5. Results in case  $\tau=0.005$  sec

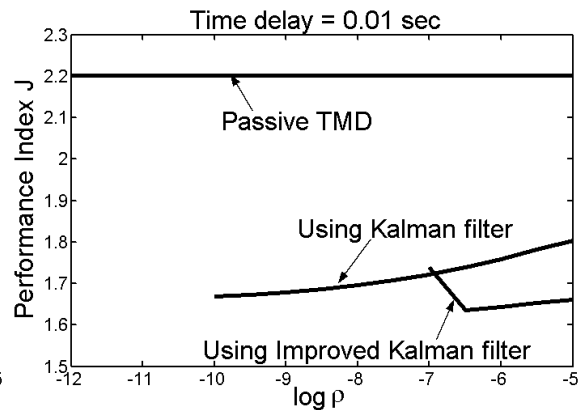


Figure 6. Results in case  $\tau=0.01$  sec

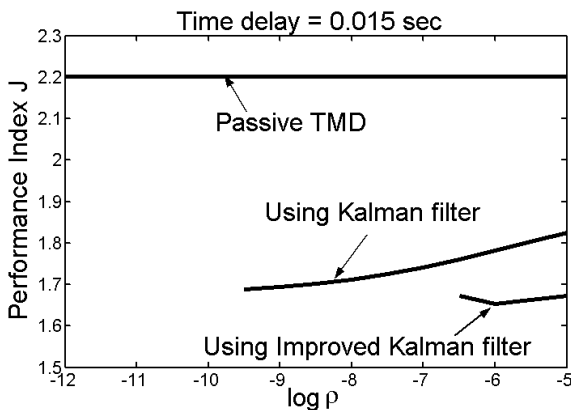


Figure 7. Results in case  $\tau=0.015$ sec

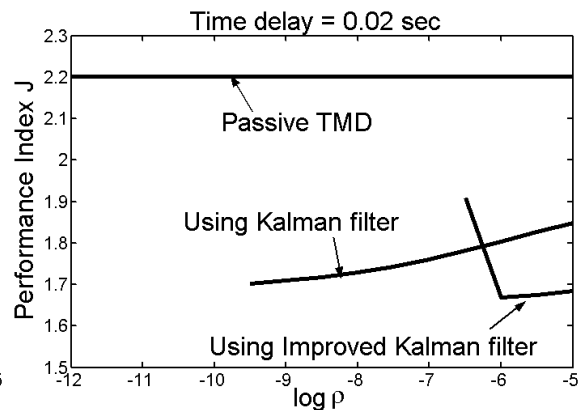


Figure 8. Results in case  $\tau=0.02$ sec

We discuss the results. It is noted that, a smaller value of  $\rho$  leads to a larger value of gain matrix  $G_e$  and also a better performance of control. In all case of time delay  $\tau$ , with the same value of  $\rho$ , the improved Kalman filter has better effect than the classical one. This implies that, the control effect could be improved without making the observer gain matrix too large. In the presence of time delay, when the gain matrix becomes too large (the value of  $\rho$  is too small), the instability occurs. The parameter  $\rho$  can not be too small because it leads to a too large gain



matrix, which can destabilize the structure. Therefore, the control using classical Kalman filter can not obtain the desired performance. Meanwhile, the control using improved Kalman filter can obtain the better performance without reducing  $\rho$ .

## 6 Conclusion

The aim of this paper is to propose an approach to improve the Kalman Bucy filter for feedback active controlled structures in presence of time delay. The improvement is achieved by adding a feedforward term to the Kalman Bucy filter. The feedforward term is chosen to eliminate the identified excitation. Using a so-called identification algorithm, the excitation is identified with a time delay and a small error term. The magnitude of error term depends on the sensor locations and the vibrational modes corresponding to large eigenvalues. To illustrate the algorithm, a numerical calculation is applied to an eight-story building subjected to earthquake ground acceleration and controlled by an active mass damper system. The presence of time delay shows that, the improved Kalman Bucy filter has good performance because it does not require a large observer gain matrix.

**Acknowledgement.** This paper was completed with the financial support of the Vietnam Basic Research Program in Natural Science.

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