

# Wavelet based Method for Remote Monitoring of Structural Health by Analysing the Nonlinearity in Dynamic Response of Damaged Structures Caused by the Crack-Breathing Phenomenon

Viet Khoa Nguyen, Oluremi A Olatunbosun, Tien Khiem Nguyen

*In this paper, a new method for remote crack monitoring, based on the crack-breathing phenomenon, is proposed. During vibration, edges of a crack come into and out of contact which causes non-linear effects in the response of a structure. This is caused by the change in the structural stiffness when the crack opens and closes. Wavelet Transform is applied to detect such nonlinear effects in the response signals of the structure. The existence and depth of the crack are determined by using large values (peaks) in the wavelet transform and modulus maxima lines of wavelet transform. The location of the crack is estimated by signal ratios between points along the structure. The results of numerical studies obtained from the FE analysis and experimental tests of simple specimens are in good agreement. The results of further numerical study of a real suspension arm based on the proposed system also promise a practical technique for remote monitoring of complex structures in practice.*

## 1 Introduction

The dynamic characteristics of a cracked structure and an intact structure are, in principle, different. The reason for this difference is the change in stiffness when a structure is cracked. During vibration a crack will open and close in time due to an externally applied loading. This phenomenon is known as the breathing process of the crack. During the breathing process the two edges of the crack come into and out of contact, thus the stiffness in the crack region may increase or decrease. This will cause changes in dynamic response of the cracked element which would be useful for detection of cracks (Nguyen V. K. and Olatunbosun O. A., 2006).

The influences of the breathing crack phenomenon on dynamic characteristics of fatigue cracked structures have been widely studied. The non-linear behaviour of the longitudinal free and forced vibration of structures was investigated using direct numerical integration (Actis R. and Dimarogonas, 1989; Chen L.W. and Chen C.L., 1988; Collin K.R., Plaut P.H., and Wauer P.H., 1992). In other researches, it has been found that the relative increase in natural frequencies due to the crack closing is much smaller than the decrease due to the crack opening (Carlson R.L., 1974; Gudmundson P., 1983). Effects of closure of cracks on dynamic responses of a cracked cantilever beam were studied using successive modal transformations (Kisa M. and Brandon J., 2000). Zastrau (1985) used the finite element method to study the steady state responses of a simply supported beam with multiple closing cracks. Non-linear distortions of the dynamics due to fatigue damage were investigated by V.V. Matveev and A.P. Bovsunovsky (2002). Methods based on change in resonant frequency are difficult to apply because the small changes of the frequency are difficult to measure. Even where these methods can be applied they cannot locate the position of the crack or determine the depth of the crack.

In the last two decades, wavelet transform has emerged as a fast-evolving mathematical and signal processing tool for many fields. The important property of wavelet transform is its capability to analyse signals locally. The position of the crack of an open cracked beam was found by applying the wavelet transform to analyse the deflection of the beam (Ovanesoova A.V. and Suarez L.E., 2004). Douka et al (2002) presented a method for crack identification in plates based on wavelet analysis. The position of the crack is determined by the sudden change in the spatial variation of the transformed displacement response. Hong et al (2002) investigated the

effectiveness of the continuous wavelet transform in terms of its capability to estimate the Lipschitz exponent as a measure of structural damage. The above researches are mode shape based and therefore impractical because of the requirement for large amounts of accurate data. A combination method of using crack breathing phenomenon and wavelet transform for local monitoring was proposed by this paper's authors (Nguyen V. K. and Olatunbosun O. A., 2006). The existence of a crack and the crack depth were determined by using wavelet transform to detect the nonlinearity of strain time history caused by the breathing crack during vibration. The position of the crack was identified by the position of the distortion in the phase shift distribution across the crack position. However, in this method, the crack site was assumed to be known or predicted.

In the studies reviewed, monitoring of responses is carried out locally (i.e. near the site of the crack) in many of the cases. In these cases, in order to detect the crack, the crack site must be known or response signals must be measured at many points on the monitored structure. However, local monitoring is difficult in practice especially if the site of a crack cannot be predicted. Moreover, for complex structures, the measurement of the response signals at a large number of points is simply impractical. Remote monitoring would therefore appear to be a more practical option. This refers to monitoring the structural response at a remote point from the site of a fatigue crack.

This study is aimed at on-line health monitoring of structures where the site of a crack is unknown or unpredicted. The structural response to be used in this method is acceleration because it can be easily measured in practice and it can be measured at any convenient position since the breathing crack influences the acceleration signal over most of the monitored structure. The theoretical background of the dynamics of a vibrating beam with a breathing crack was described in a previous paper (Nguyen V. K. and Olatunbosun O. A., 2006) along with the application of wavelet transform in crack detection. The current paper will present the results of a numerical and experimental study to establish a technique for remote monitoring of fatigue cracks.

**2 Numerical Studies**

*a. Simulation of cracked beam vibration response*

In order to analyse the dynamic response of a cracked beam, ALGOR - finite element software has been used. The crack is described as shown in Figure 1. In this model the crack consists of two close edges which may come in and out of contact during vibration. It is expected that when the load is a sinusoidal function the acceleration response function should not be purely sinusoidal but it should be distorted due to the non-linear effects of the crack opening and closing.

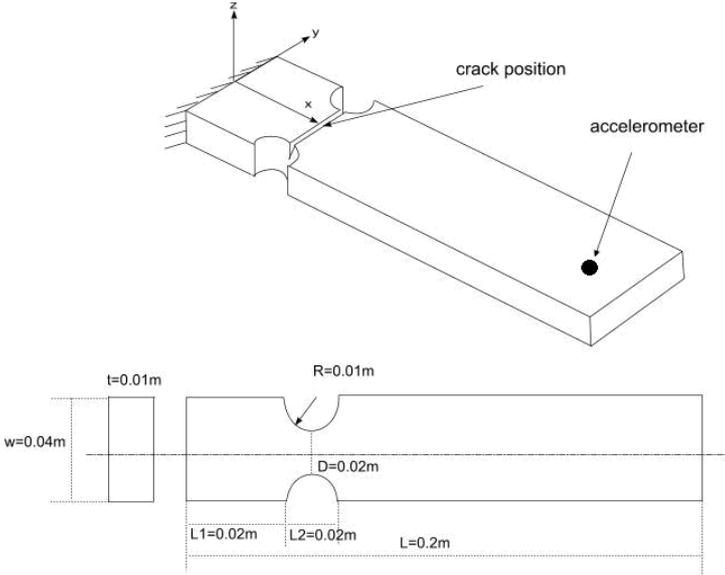


Figure 1. A cantilever beam with a crack

Eight levels of the crack from 0 to 60% have been investigated. These eight cases are numbered as in Table 1.

Case	1	2	3	4	5	6	7	8
Crack depth (%)	0	1	10	20	30	40	50	60

Table 1. Eight cases with cracks of varying depths at crack position  $x=30$  mm

**b. Detection of crack existence and crack depth using wavelet transforms**

The wavelet transform is defined as follows [Daubechies 1992]:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi^* \left( \frac{t-b}{a} \right) dt \tag{1}$$

Where  $f(t)$  is input signal;  $a$  is a real number called scale or dilation;  $b$  is a real number called position;  $W(a,b)$  are wavelet coefficients at scale  $a$  and position  $b$ ;  $\psi \left( \frac{t-b}{a} \right)$  is wavelet function and  $\psi^* \left( \frac{t-b}{a} \right)$  is complex conjugate of  $\psi \left( \frac{t-b}{a} \right)$ .

- *Detection of crack existence*

The acceleration signals are measured at the free end of the beam. Figures 2 to 4 present the normalized acceleration-time history and its continuous wavelet transform for each of three different levels of the crack. In these figures, the upper graph is the acceleration signal and the lower graph is its wavelet transform. Figure 2 indicates no discontinuity in the wavelet transform when the beam is intact. Figures 3 and 4 show increasing levels of discontinuity in the wavelet transform of the acceleration signal as crack depth increases, clearly indicating distortions in the acceleration signals at moments that the crack opens or closes.

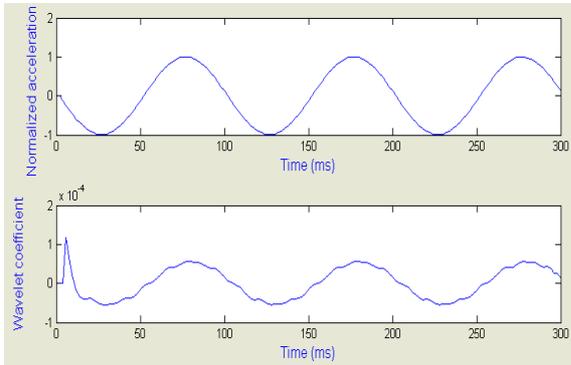


Figure 2. Crack depth is 0%

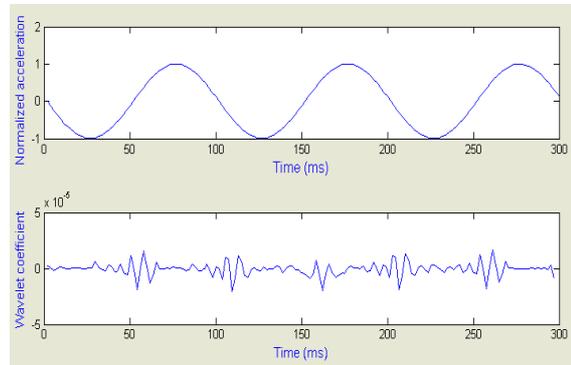


Figure 3. Crack depth is 1%

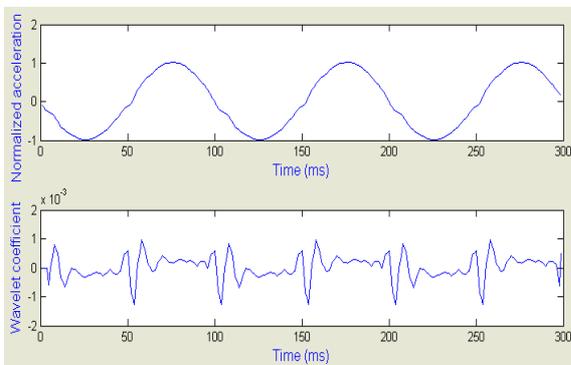


Figure 4. Crack depth is 20%

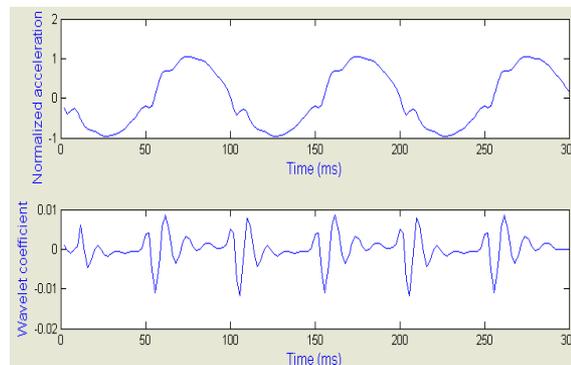


Figure 5. Crack depth is 60%

- *Detection of crack depth*

The crack depth is determined using the modulus maxima lines of the wavelet transform. Mallat (1992) gave a definition of local maxima of wavelet transform as follows:

- Local extremum is any point  $(a_0, t_0)$ , such that  $\frac{\partial W(a_0, t)}{\partial t}$  has zero-crossing at  $t=t_0$  when  $t$  varies.
- Modulus maxima is any point  $(a_0, t_0)$  such that  $|W(a_0, t)| < |W(a_0, t_0)|$  when  $t$  belongs to either the right or the left neighbourhood of  $t_0$ , and  $|W(a_0, t)| \leq |W(a_0, t_0)|$  when  $t$  belongs to the other side of the neighbourhood of  $t_0$ .
- Maxima line is any connected curve in the scale space  $(a, t)$  along which all points are modulus maxima.

According to Mallat (1992)

$$\log_2(|W(a, b)|) \leq \log_2 A + h \log_2(a) \quad (2)$$

where  $A$  is constant of the modulus maxima line and  $h$  is Lipschitz exponent of the wavelet transform. Using this equation the crack depth can be calculated. In practice, if different signals have the same Lipschitz exponent at a discontinuous point, the discontinuities may have the same physical cause. In this case, from Equation (2), parallel lines describing the relationship between  $\log_2(|W(a, t)|)$  and  $\log_2(a)$  with the same gradient but different intercept  $\log_2(A)$  can be constructed. Therefore, the intercept of these parallel lines, in some cases, relates to the level of the discontinuity or the level of the cause at the given point. In this study, discontinuities of signals caused by breathing cracks of the same types but with different crack depths are investigated. If the Lipschitz exponent of the signals remains the same then the different intercepts of parallel lines of Equation (2) can discriminate the different levels of crack depth.

Figure 6 shows that when the Lipschitz exponent  $h$  is fixed, only the intercept  $\log_2(A)$  changes when the crack depth changes. Thus, each parallel line is distinguished by its intercept  $\log_2(A)$ . This intercept increases when crack depth increases. Because of this, the intercept  $\log_2(A)$  can be considered as an intensity factor which relates the extent of the fatigue crack to the wavelet coefficients. Establishing a graph of intercept  $\log_2(A)$  versus crack depth from Figure 6, a relationship between intercept  $\log_2(A)$  (or intensity factor) and crack depth is obtained as shown in Figure 7. It can be seen that this relationship is a line.

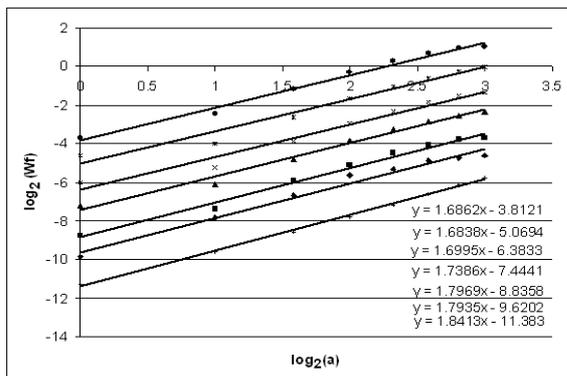


Figure 6. Maxima lines versus scale for seven levels of crack: 1% to 60%

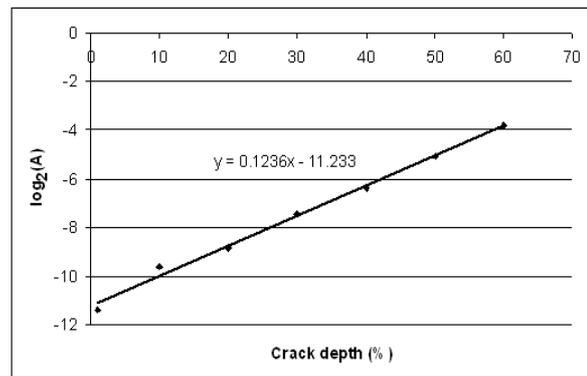


Figure 7: Intensity factor versus crack depth

- *Detection of crack position using signal ratio*

For a structure with a crack, it is considered that the structure consists of two different sections joined at the crack position. Detection of the crack position is based on the change in the stiffness of the cross-section at the crack when it breathes. During vibration, the signal ratio of normalized acceleration between two points on the same section of the structure is expected to have little or no distortion, whereas the signal ratio between two points on the two different sections, either side of the crack, should have more significant distortion due to the change in cross-sectional stiffness at the crack as the crack opens and closes.

The signal ratio between two points can be calculated as follows

$$g(t) = \frac{f_1(t)}{f_2(t)} \quad (3)$$

where  $g(t)$  is the signal ratio;  $f_1(t)$  is the response signal at point 1 and  $f_2(t)$  is the response signal at point 2.

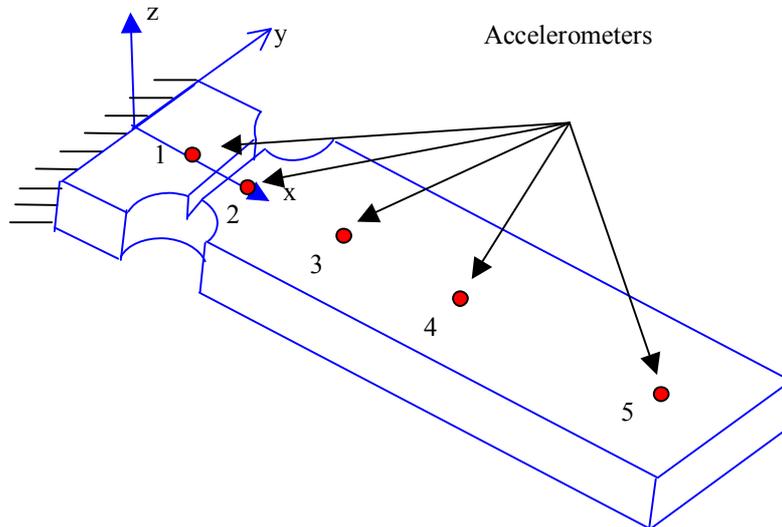


Figure 8. Positions of five measurement points along the specimen

Using data from FE analysis, seven levels of crack depth (see Table 1) are applied to investigate the influence of the position of the crack on the signal ratio between different points. In each case, five points along the specimen are selected to measure the signal ratio as can be seen in Table 2 and Figure 8. Signal ratios are calculated between point 5 (fixed) and the other four points 1, 2, 3, 4. The crack position is at  $x=30\text{mm}$ , i.e. in between point 1 and point 2.

Point	1	2	3	4	5
x (mm)	13	36	47	108	170

Table 2. Positions of measurement points

The results show that the signal ratios between points 1, 2, 3, 4 and point 5 are constant and equal to 1 for the intact specimen. This implies that when there is no crack, there is no factor to influence the transmission of vibration signals along the specimen.

When there is a crack, the signal ratios between points 2, 3, 4 and point 5 on the same section are constant 1 while the signal ratios between two points on different sections, i.e. between point 1 on section I and point 5 on section II, have larger values compared with signal ratios of points on section II. The signal ratios from point 1 to point 4 show that there are significant changes in the signal ratio when it passes from point 1 to point 2. This means that the crack position is in the area between point 1 and point 2 as expected.

### 3 Experimental Results

Having developed the remote monitoring method based on the combination of breathing crack and wavelet transform as described in the previous sections, it is very important to demonstrate its practical application for structural health monitoring. For this purpose, experimental tests have been carried out to detect and monitor fatigue cracks in beam structures similar to that used in the numerical studies. Structural response was monitored in the form of acceleration response signals from which the health of the structure could be determined at various stages of fatigue damage.

**a. Detection of crack existence**

Figures 9 to 12 present normalized acceleration-time histories and their wavelet transforms for four cases taken at different stages during the fatigue test on the specimen. They show increasing levels of discontinuity in the wavelet transform of the acceleration signal as crack depth increases, clearly indicating distortions in the acceleration signals at moments when the crack opens or closes. This indicates that the experimental result of crack detection using acceleration is in agreement with the FE analysis result.

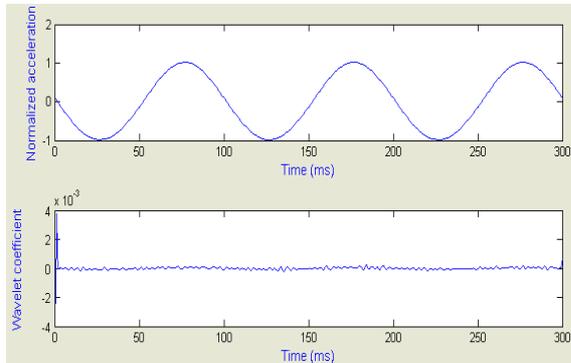


Figure 9. Crack depth is 0%

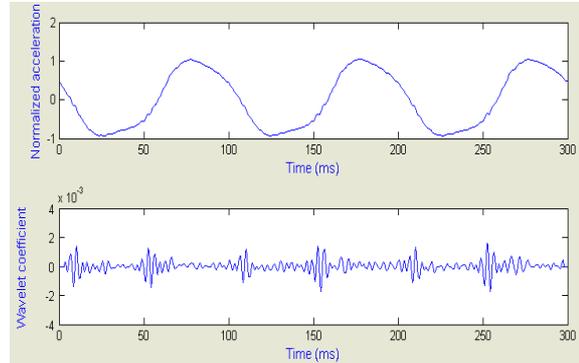


Figure 10. Crack depth is 20.1%

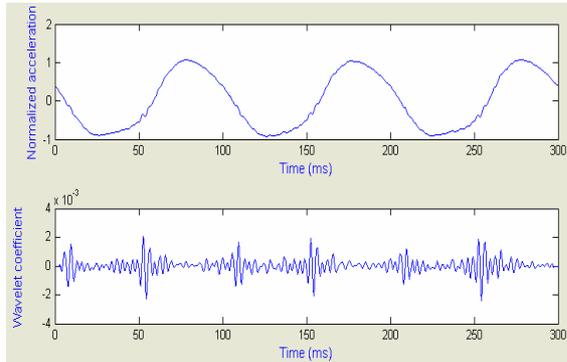


Figure 11. Crack depth is 26.3%

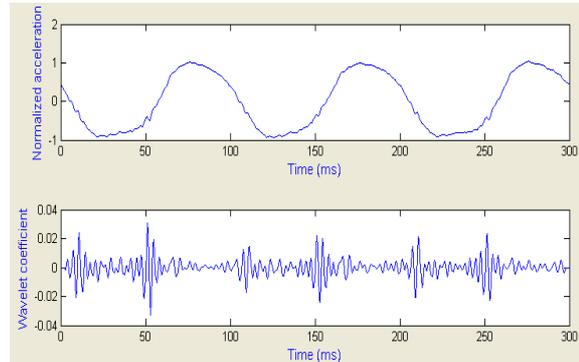


Figure 12. Crack depth is 41.5%

**b. Detection of crack depth**

The relationship between intercept  $\log_2(A)$  (or intensity factor) and crack depth, obtained from the modulus maxima lines for the different crack depths is shown in Figure 16. It shows a linear relationship between the intercept  $A$  and crack depth in a similar manner to the numerical study results.

Figure 17 presents the comparison between experimental and FE analysis results of the relationship between the intercept  $A$  and the crack depth. The figure shows close agreement between the numerical and experimental results. However there are minor discrepancies. The Lipschitz exponent from the experiment is 1.86 and is different from FEA where the value of Lipschitz is 1.72. This can be explained by the fact that in the FE analysis the crack is modelled as surface to surface contact without rebounding, sticking and sliding phenomena which may occur in practice. Moreover, the surfaces of two edges of crack are not as smooth as modeled in FE analysis. Background noise may also be a contributory factor to the difference between numerical and experimental results.

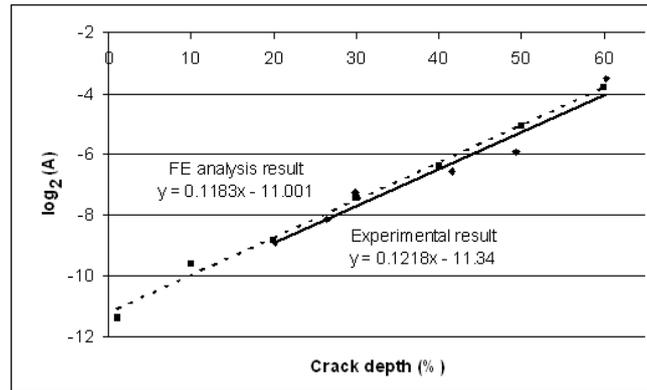


Figure 13. Relationship between crack depth and intercept:  
the dotted line is the FE analysis result;  
the other line is the experimental result

### c. Detection of crack position

The experimental results are quite in agreement with the numerical results: for each crack depth greater than 0, the signal ratio has a significant change when it goes across from point 1 to point 2 and then remains nearly the same from point 2 to point 4. This means that the crack is in between point 1 and point 2.

## 4 Application to a Real Component

The new technique has been applied to a simple beam structure and has given promising results. In this section further studies are carried out on a real component in order to establish the application of the developed method in practice. Figure 14 shows the finite element model of a suspension arm. Figures 15 to 18 show the detection of crack existence using wavelet transform. Figure 19 presents the relationship between the intensity factor versus crack depth. Figure 20 describes the configuration of measurement points along the suspension arm structure. By using the signal ratio the position of the crack is found in between point 2 and point 3. This result is in agreement with the finite element model of the suspension arm. These results verify the effectiveness of the new technique for assessment and monitoring the integrity of structures.

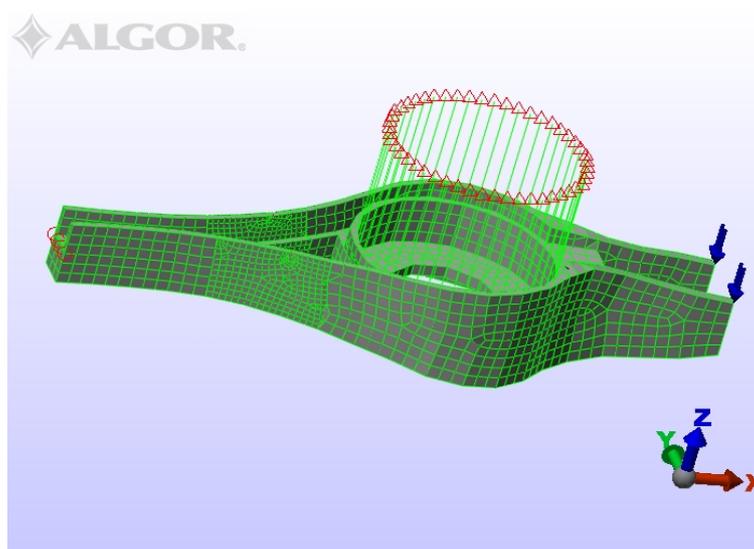


Figure 14. Suspension arm of a car

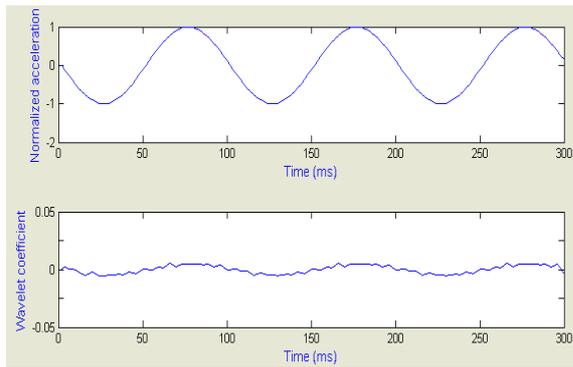


Figure 15. Crack depth is 0%

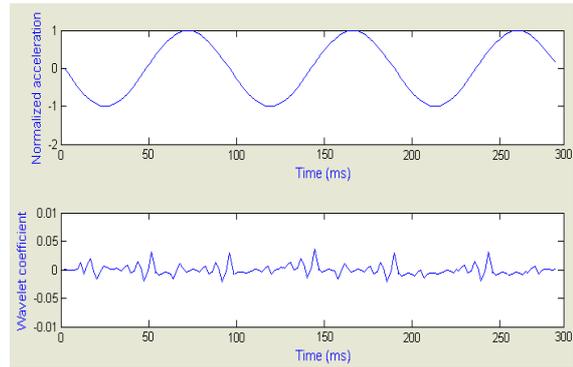


Figure 16. Crack depth is 1%

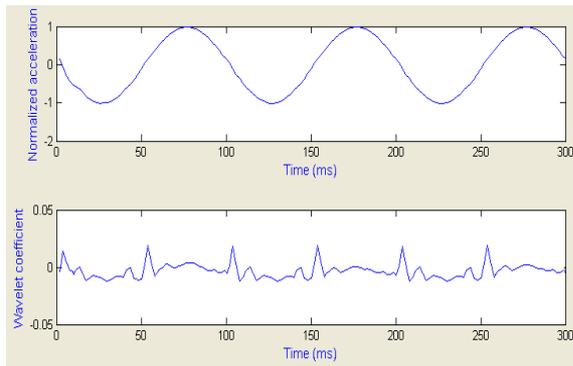


Figure 17. Crack depth is 20%

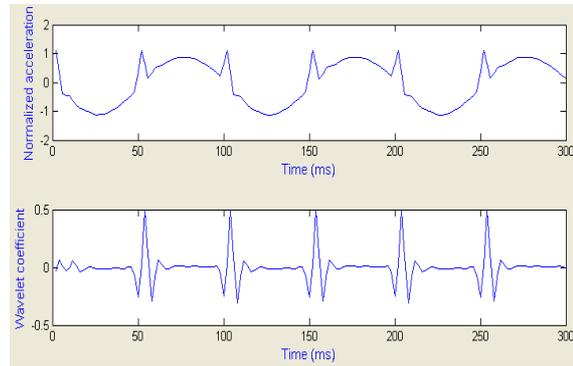


Figure 18. Crack depth is 60%

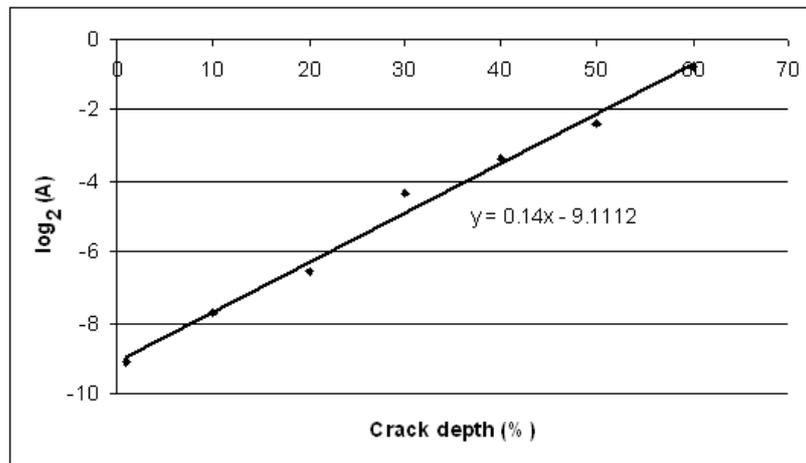


Figure 19. Intensity factor versus crack depth calculated from acceleration signals

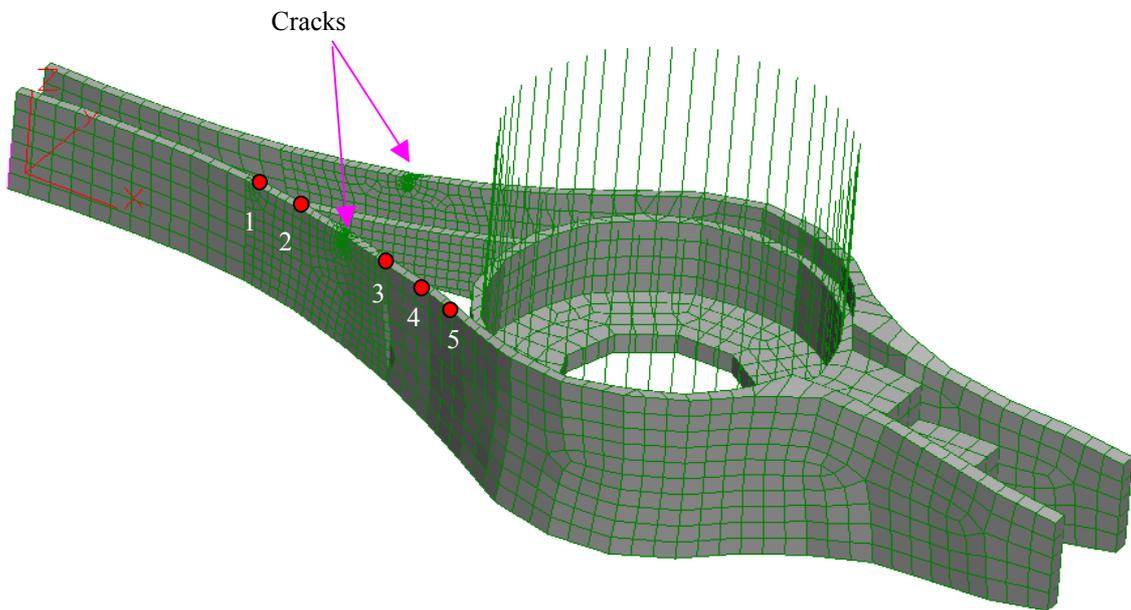


Figure 20. Positions of five measurement points along the component

#### 4 Conclusions

A practical method for remote monitoring of a fatigue crack has been proposed. In this method, the acceleration response signal is used because it can be easily measured in practice and the acceleration response over most of the monitored structure can be influenced by the crack-breathing phenomenon. The method is a development of the authors' previous study based on the combination of crack breathing and wavelet transform to detect the existence of a crack in a structure and to determine the crack depth. A new approach based on a signal ratio analysis for detection of the crack position is also proposed. Detection of the crack position is based on the significant change in the signal ratio between two points on either side of the crack due to the change in cross-sectional stiffness at the crack as the crack opens and closes.

The method has been demonstrated by a numerical study and a fatigue test on a simple specimen carried out in the laboratory. The appearance of the crack detected from experimental data is in agreement with the result of FE analysis. The relationship between crack depth and the intensity factor  $\log_2(A)$  is established and can be used for crack depth estimation. However, this relationship depends on structures and position of signals. Therefore, in order to use this relationship to estimate the crack depth, numerical analyses need to be done in advance. The ability of the new approach to detect the crack position, based on the signal ratio, has been established in the numerical study and confirmed experimentally.

The results obtained from the numerical experiments on a real suspension arm of a car point out that the developed method can be applied not only to a simple specimen, but it can also be applied for more complicated structures.

In conclusion, the new method can be used for detection and remote monitoring of a crack in structures. The advantages of this technique are that it is capable of being implemented in a practical manner since the acceleration time history signal can be easily measured in practice. Only one acceleration time history is needed to detect the existence of the crack as well as to establish the relationship between the intensity factor with the crack depth and only few measurements are needed to estimate the crack site. While in mode shape based methods, a large amount of quality signals are required for the damage detection problem. Furthermore, the use of acceleration signal facilitates remote monitoring of the structure for fatigue cracks when the crack site is

unknown. From examples investigated in this study, the proposed method can be applied to detect small cracks with the lengths of 1% of the cross section. Obviously, the proposed method is more sensitive to frequency based methods since natural frequencies are almost constant up to the crack depth of 50% and then they slowly decrease (Carlson, 1974 and Gudmunston, 1983).

The proposed method is based on the crack breathing phenomenon caused by a surface crack. Other defects, for instance internal defects that do not produce crack breathing phenomenon therefore are out of the scope of the paper.

This paper has established the proposed technique for remote structural health monitoring using a simple structure and further studies on real components under real test conditions are proposed to further validate the method.

## References

- Nguyen V. K., Olatunbosun O. A., *A proposed method for fatigue crack detection and monitoring using the breathing crack phenomenon and wavelet analysis*. Journal of Mechanics of Materials and Structures 2007, Vol 2 No 3, p. 400-420.
- Actis R. and Dimarogonas., *Non-linear Effects Due To Closing Cracks in Vibrating Beams*. 12<sup>th</sup> ASNE Conference on Mechanical Engineering, Vibration and Noise, Montreal, Canada, September 1989, Vol. 18, 99-104.
- Chen L.W. and Chen C.L., *Vibration and Stability of Cracked Thick Rotating Blades*. Computers and Structures 1988, Vol. 28, p. 67-74.
- Collin K.R., Plaut P.H., and Wauer P.H., *Free and Forced Longitudinal Vibrations of a Cantilevered Bar with Crack*. ASME Journal of Vibration and Acoustics 1992, Vol. 114, p. 171-177.
- Carlson R.L., *An Experimental Study of the Parametric Excitation of a Tensioned Sheet with a Crack-like Opening*. Experimental Mechanics 1974, Vol. 14, p. 52-458.
- Gudmundson P., *The Dynamic Behaviour of Slender Structures with Cross-sectional Cracks*. Journal of Mechanics Physics Solids 1983, Vol. 31, p. 329-345.
- Kisa M. and Brandon J., *The Effects of Closure of Cracks on the Dynamics of a Cracked Cantilever Beam*. Journal of Sound and Vibration 2000, Vol. 238 (1), p. 1-18
- Zastrau B., *Vibration of Cracked Structures*. Archive of Mechanics 1985, Vol. 37, p. 731-743.
- Matveev V.V. and Bovsunovsky A.P., *Vibration-based Diagnostics of Fatigue Damage of Beam-like Structures*. Journal of Sound and Vibration 2002, Vol. 249 (1), p. 23-40.
- Ovanesoava A.V. and Suarez L.E., *Application of Wavelet Transform to Damage Detection in Frame Structures*. Journal of Engineering Structure 2004, Vol. 26, p. 39-49.
- Douka E., Loutridis S., Trochidis A., *Crack Identification in Plates Using Wavelet Analysis*. Journal of Sound and Vibration 2002, Vol. 249 (1), p. 23-40.
- Hong J.-C., Kim Y.Y., Lee H.C., and Lee Y.W., *Damage Detection Using the Lipschitz Exponent Estimated by the Wavelet Transform: Applications to Vibration Modes of a Beam*. International Journal of Solids and Structures 2002, Vol. 39, p. 1803-1816.
- Daubechies I., *Ten lectures on wavelets*. CBMS-NSF Conference series, 61. Philadelphia, PA: SISAM, 1992.
- Mallat S., Hwang W., *Singularity Detection and Processing with Wavelets*. IEEE Transactions of Information Theory 1992, Vol. 38, pp. 617-643.

---

*Address:* Dr. Nguyen Viet Khoa, Prof. Dr. of Sc. Nguyen Tien Khiem, Institute of Mechanics, Vietnamese Academy of Science and Technology, 264 Doi Can Street, Ba Dinh, Hanoi, Vietnam  
email: nvkhoa@imech.ac.vn; ntkhiem@imech.ac.vn