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Damping Identification Using the Wavelet-based Demodulation Method: Application to Gearbox Signals

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The identification of damping in vibrational systems from measured free responses is a well-known problem and remains a challenge to dynamics specialists. Compared to an estimation of the stiffness, the damping coefficient or, alternatively, the damping ratio is the most difficult quantity to evaluate. In this paper, a novel procedure is developed for the identification of modal damping ratios in a multi-degree-of-freedom vibrational system . The procedure is based on the demodulation technique with respect to the continuous wavelet transform and the Morlet-wavelet function. Numerical simulations demonstrating the applicability of the method to multi-degree-of-freedom vibrational systems are presented. The proposed procedure is also compared with measured response data from a gearbox.

1 Introduction

The identification of damping in vibrational systems from measured response data appears to be of crucial interest. Compared to an estimation of the stiffness, the damping coefficient or, alternatively, the damping ratio is the most difficult quantity to determine. Damping always requires a dynamic test to measure. The majority of damping measurements performed today are based on experimental modal analysis. However, this approach requires a specific measurement hardware and a complex software for determining the frequency response function of the system and extracting the modal data. Furthermore, the frequency response functions will often give significant errors resulting from influences of the noise.

From a signal processing viewpoint, measured free responses of the vibrational system are usually effected by the modulation phenomena. The response signal is the product of the modulating signal with the high-frequency carrier signal. The modulation signal is the signal envelope which measures the dissipation of vibration energy caused by damping, and the carrier signal contains some resonance frequencies of the vibrational system . In order to identify the damping directly from this response signal, the signal envelope must be extracted by using specific demodulation methods. The Hilbert transform, combined with the digital filtering technique and FFT, is usually used as a traditional method for extracting the signal envelope. However, this method exhibits several weaknesses due to the inconvenient selection of the passband and the spectrum overlap.

Over the past 10 years, wavelet theory has become one of the emerging and fast-evolving mathematical and signal processing tool for its many distinct merits (Mallat, 1999). General overview of application of the wavelet transform may be found in some studies, e.g. (Peng and Chu, 2004), (Nguyen Phong Dien, 2003). The wavelet transform has dominant advantages in signal filtering and time-frequency characteristics, which make it applicable to system identification (Robertson et. al, 1998), (Newland, 1999). The state-of-the-art demodulation method is based on the continuous wavelet transform (CWT). Yuh-Tay Sheen and Chun-Kai Hung (2004) proposed a novel wavelet-based envelope function for demodulating vibration signals. This envelope function is effectively used to extract the envelope of the signal. The study showed that the CWT of the signal simultaneously provides the modal extraction and the signal envelopes for estimating damping ratios.

Recently, the common CWT-based procedures for damping identification in multi-degree-of-freedom vibrational systems are already known, e.g. (Ruzzene et. al., 1997). Staszewski (1997) introduced a effective method for estimating the damping ratios by using CWT and the Morlet-wavelet function. Boltezar and Slavic (2004) proposed a procedure using the modified CWT and the Gabor-wavelet function to overcome a some limitations of Staszewski's method, such as the edge-effect.

In this paper, a novel procedure is developed for the identification of modal damping ratios in a multi-degree-of-freedom vibrational system . Basically, the procedure is based on the demodulation technique with respect to the wavelet ridges of the CWT and the Morlet-wavelet function. Some extensions of the method will be proposed. The following numerical example demonstrates the improvements of the proposed wavelet-based method to damping identification. The procedure was also tested by measured response data from a gearbox.

2 The Continuous Wavelet Transform and Wavelet Ridges

This section presents a brief background on the Continuous Wavelet Transform (CWT) utilized in this paper. A thorough discussion of wavelets is given in several references, e.g. (Mallat, 1999).

Using a mother-wavelet function $\psi(t)$, wavelet functions are family of functions of type

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right), \quad \tau, s \in \mathbf{R}^+ \tag{1}$$

generated from $\psi(t)$ by the operation of dilation by s and translation by τ. In the time domain, $\psi_{\tau,s}$ is centered at τ with a spread proportional to s. The CWT of a signal $x(t)$ is defined by

$$\mathbf{CWT}\{x(t)\} = \mathrm{W}_x(\tau,s) = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{2}$$

where ψ^* is the complex conjugate of the wavelet function ψ. A wavelet coefficient $\mathrm{W}_x(\tau,s)$ measures the variation of signal $x(t)$ in a neighborhood of position τ. By varying the scale s and translating along the localized time τ, the wavelet modulus $\left|\mathrm{W}_x(\tau,s)\right|$ can construct a wavelet plot showing both the amplitude of any features in the signal versus the scale and how this amplitude varies with time. Therefore, the information in the time domain will still remain, in contrast to the Fourier Transform (FT), where the time domain information becomes almost invisible after the integration over entire record length of the signal

$$\mathbf{FT}\{x(t)\} = \hat{x}(\omega) = \int_{-\infty}^{+\infty} x(t)e^{i\omega t}dt \tag{3}$$

A complex mother-wavelet function can be constructed with a frequency modulation of a real and symmetric window $g(t)$

$$\psi(t) = g(t)e^{i\eta_0 t} \tag{4}$$

where η_0 is a constant parameter and $i = \sqrt{-1}$. The Morlet mother-wavelet is obtained with a Gaussian window

$$g(t) = \pi^{-1/4}\ e^{-t^2/2} \tag{5}$$

$$\hat{g}(\omega) = \pi^{-1/4}\ e^{-\omega^2/2} \tag{6}$$

In this study the Morlet mother-wavelet is used to perform the CWT. The relation between the scale s and the frequency ω of the Morlet wavelet function can be derived analytically as (Torrence and Compo, 1998)

$$s = \frac{\eta_0}{\omega} \tag{7}$$

The parameter η_0 balances the time-frequency resolution of the Morlet wavelet. Increasing the value of η_0 will increase the frequency resolution, but it decreases the time resolution (Meltzer and Nguyen Phong Dien, 2004).

Consider a signal $x(t)$ that can be expressed by a modulated sinusoidal function

$$x(t) = a(t)\cos(\alpha_0 t + \beta) = a(t)\cos\varphi(t) \tag{8}$$

where $a(t)$ is the envelope function of $x(t)$ and $\varphi(t) = \alpha_0 t + \beta$, α_0 and β are constant.

The CWT of this signal can be derived analytically by a simple function as follows (see also Mallat, 1999)

$$W_x(\tau,s)=\frac{\sqrt{s}}{2}a(\tau)e^{i\varphi(\tau)}\hat{g}\left(\eta_0-s\alpha_0\right)+\varepsilon(\tau,s) \qquad (9)$$

where $\hat{g}(\eta_0-s\alpha_0)$ is the Fourier transform of $g(t)$ at $\omega=\eta_0-s\alpha_0$ and $\varepsilon(\tau,s)$ is the corrective term. If the variations of $a(t)$ are slow compared to the period $2\pi/\alpha_0$, the term $\varepsilon(\tau,s)$ can be neglected (Mallat, 1999). In this case, by considering relationship (7) the CWT of $x(t)$ can be rewritten in the form

$$W_x(\tau,\omega)=\frac{1}{2}\sqrt{\frac{\eta_0}{\omega}}\;a(\tau)e^{i\varphi(\tau)}\hat{g}\left(\eta_0-\frac{\eta_0\alpha_0}{\omega}\right) \qquad (10)$$

The wavelet modulus $\left|\mathrm{W}_x(\tau,s)\right|$ is given by

$$\left|W_x(\tau,\omega)\right|=\frac{1}{2}\sqrt{\frac{\eta_0}{\omega}}\;a(\tau)\left|\hat{g}\left(\eta_0-\frac{\eta_0\alpha_0}{\omega}\right)\right| \qquad (11)$$

Since $\left|\hat{g}(\omega)\right|$ is maximum at $\omega=0$, expression (11) shows that $\left|\mathrm{W}_x(\tau,\omega)\right|$ gets the maximal value at $\omega=\alpha_0$. The corresponding points $\left(\tau,\alpha_0\right)$ are called wavelet ridges. A set of these points creates a parallel line with the time axis in the wavelet plot.

For $\omega=0$ expression (6) leads to $\hat{g}(0)=\pi^{-1/4}$. The envelope $a(\tau)$ at $\omega=\alpha_0$ is given by

$$a(\tau)=2\pi^{1/4}\sqrt{\alpha_0/\eta_0}\;\left|W_x(\tau,\alpha_0)\right|=\mathrm{K}.\left|W_x(\tau,\alpha_0)\right| \qquad (12)$$

where K is a positive constant, $\mathrm{K}=2\pi^{1/4}\sqrt{\alpha_0/\eta_0}$. Expression (12) shows that the wavelet modulus $\left|\mathrm{W}_x(\tau,\omega)\right|$ at wavelet ridges (i.e. $\omega=\alpha_0$) is proportional to the envelope of signal $x(t)$ described in expression (8). $\left|\mathrm{W}_x(\tau,\alpha_0)\right|$ are called the wavelet envelope of the signal $x(t)$ at frequency α_0.

3 Damping Ratio Estimation Procedure

It is well known that a damped n-degree-of-freedom system with proportional damping

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{B}\dot{\mathbf{q}}+\mathbf{C}\mathbf{q}=\mathbf{0} \qquad (13)$$

can be solved by using the modal analysis. This leads to n decoupled modal equations

$$\ddot{p}_i(t)+2\zeta_i\omega_{0i}\dot{p}_i(t)+\omega_{0i}^2p_i(t)=0,\quad i=1,2,...,n \qquad (14)$$

Here $p_i(t)$ denotes the i-th modal coordinate, ω_{0i} is the i-th undamped natural frequency and ζ_i is the i-th modal damping ratio. The modal damping places an energy dissipation term of the form $2\zeta_i\omega_{0i}\dot{p}_i(t)$ (viscous damping). This form is chosen largely for its mathematical convenience. The modal damping ratios ζ_i are assigned by making measurements of the free damped response and estimating ζ_i.

As can be seen from expression (2), the CWT is a linear transform. Its linearity makes it possible to analyze each i-th component $x_i(t)$ of a multi-component signal

$$\mathbf{CWT}\left\{\sum_{i=1}^{n}x_i(t)\right\}=\sum_{i=1}^{n}\mathbf{CWT}\left\{x_i(t)\right\} \qquad (15)$$

The task of interest is to determine the modal damping ratios associated with each mode shape. The linearity of the CWT is useful for extracting modal data of each mode from measured free responses of the system.

Consider the free response of a linear vibrational system for the unterdamped case, the signal model corresponding to the i-th mode can be described by

$$x_i(t)=A_ie^{-\zeta_i\omega_{0i}t}\sin(\omega_{di}t+\varphi_i) \qquad (16)$$

where A_i is a constant amplitude, ω_{0i} and ω_{di} are the i-th undamped and damped natural frequency respectively. The decay envelope of this signal is $a_i(t)=A_ie^{-\zeta_i\omega_{0i}t}$. Note that it is difficult to extract the envelope $a_i(t)$ from measured free response with numerous natural frequencies. According to the signal model (16), the logarithmic decrement δ_i is given by

$$\delta_i = \frac{1}{m} \ln \frac{a_i(t)}{a_i(t + mT_i)} \tag{17}$$

where m is any positive integer and $T_i = 2\pi / \omega_{di}$. Note that the variations of $a_i(t)$ are slow compared to the period T_i. By using expressions (12) and (17), the logarithmic decrement δ_i can be expressed in term of wavelet modulus as

$$\delta_i = \frac{1}{m} \ln \frac{|W_{xi}(t, \omega_{di}|}{|W_{xi}(t + mT_i, \omega_{di})|} \tag{18}$$

Finally, the modal damping ratio ζ_i can then be determined from the value of δ_i (Nguyen Phong Dien, 2005)

$$\zeta_i = \frac{\delta_i}{\sqrt{4\pi^2 + \delta_i^2}} \tag{19}$$

A specialized program has been developed on the MATLAB® numeric computing environment for estimating damping ratios from measured free responses of vibrational systems. The program includes the following procedure:

- transforming the time signal into time-frequency domain by using the CWT
- detecting wavelet ridges in the wavelet plot to estimate natural frequencies ω_{di}
- extracting the wavelet envelope $|\mathrm{W}_x(\tau, \omega)|$ at the natural frequencies
- calculating the damping ratios according to formulas (18) and (19).

4 Numerical Example

In order to assess the performance of the proposed wavelet-based demodulation method, a test-signal with three exponentially-decaying components was chosen for simulating the free response of a damped vibrational system

$$x(t) = r(t) + \sum_{i=1}^{3} a_i e^{-\zeta_i \omega_i t} \sin(\omega_i t + \varphi_i), \tag{20}$$

where $r(t)$ is a random signal which describes the noise. The parameters of the signal are given in Table 1. The frequencies of signal-components are very different, the last component has a high frequency with a small damping ratio. Figure 1 shows the signal in the time domain.

Component i	a_i	$f_i = \omega_i / 2\pi$ (Hz)	ζ_i	φ_i
1	1.0	250	0.02	$-\pi/8$
2	1.25	500	0.045	$\pi/6$
3	0.5	1250	0.005	$\pi/8$

Table 1. Parameters of the test-signal

Firstly, the signal is transformed in time-frequency domain using the CWT with Morlet-wavelet. Figure 2 is the wavelet plot displayed as a three-dimensional surface, obtained by plotting the amplitude of the wavelet modulus $|\mathrm{W}_x(\tau, \omega)|$ against time and frequency. Three exponentially-decaying components are separated from the original signal and can be clearly identified.

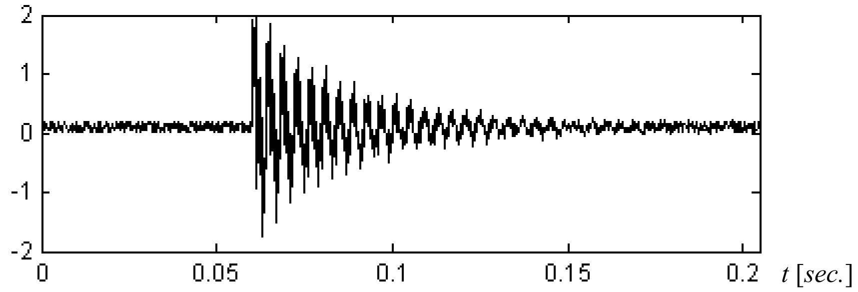

Figure 1. The test-signal

In order to identify the “natural” frequency ω_i of each signal-component from the wavelet plot in Figure 2, a numerical algorithm is developed to determine wavelet ridges. The positions of the wavelet ridges reveal the corresponding natural frequencies in frequency axis (see section 3).

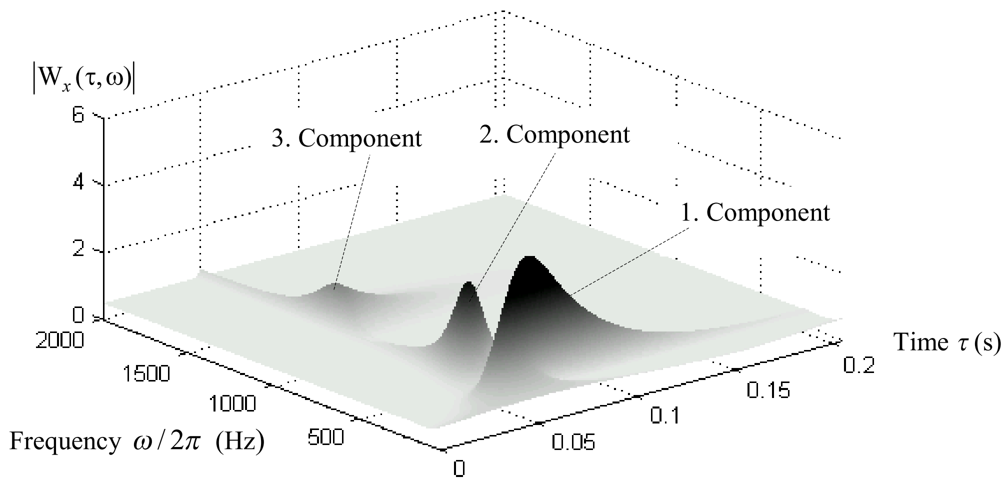

Figure 2. The wavelet plot of the test-signal

The wavelet envelope $|W_x(\tau,\omega_i)|$ can be extracted from the wavelet plot by a slice parallel to the time axis through frequency ω_i in the frequency axis. According to formulas (18) and (19), this wavelet envelope can be used to estimate the corresponding damping ratio. Figure 3 shows the wavelet envelope extracted at the third natural frequency ω_3. The identification data are given in Table 2. It can be seen that results of the identification task are very good.

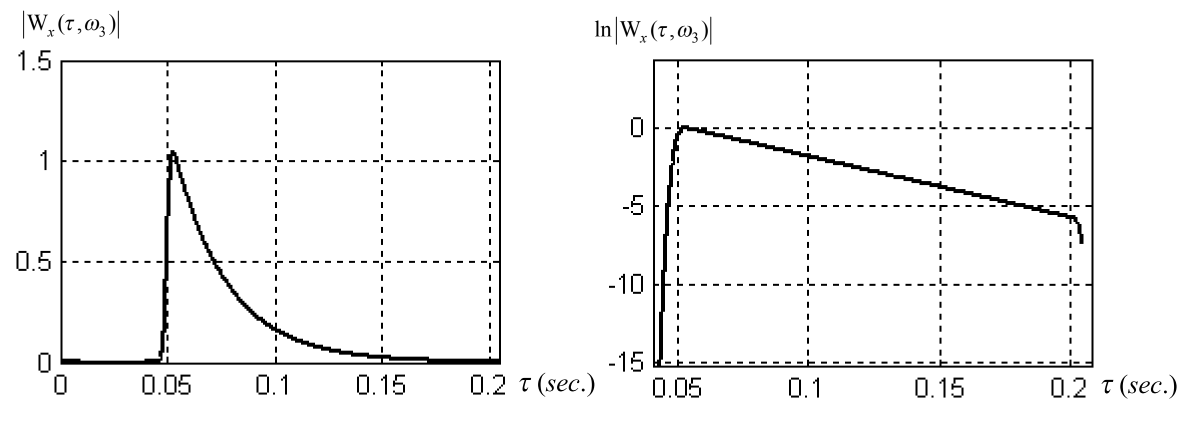

Figure 3. The wavelet envelope of the third signal-component extracted from the wavelet plot in Figure 2

Parameter	$i = 1$	$i = 2$	$i = 3$
$\omega_i / 2\pi$ (Hz)	247	500	1255
T_i (*sec.*)	4×10^{-3}	2×10^{-3}	7.97×10^{-4}
m	10	15	50
τ_0	0.080	0.0567	0.060
$\lvert W_x(\tau_0, \omega_i)\rvert$	2.6167	2.5620	0.9835
$\lvert W_x(\tau_0 + mT_i, \omega_i)\rvert$	0.7340	0.0485	0.2060
δ_i	0.1271	0.2645	0.0313
ζ_i	0.0202	0.0421	0.0049
Error of ω_i (%)	1.1	0.0	0.4
Error of ζ_i (%)	1.0	6.4	2.0

Table 2. Results of damping identification for the test-signal

5 Experiment

Now the above proposed method is applied to estimate the damping ratio of a spur gear-pair system. The dynamical model used for parameter identification has been presented in (Nguyen Van Khang et. al., 2004). The gear mesh is modelled as a pair of rigid disks connected by a spring-damper set along the line of contact (see also details in Appendix A).

The experiments have been performed at an ordinary back-to-back test rig to investigate the influence of different tooth faults on the measured vibration signals and the feasibility of an early fault recognition by some signal processing methods. The test rig and the test procedure is described in detail in some studies, e.g. Nguyen Phong Dien (2003), Meltzer and Nguyen Phong Dien (2004). Only some specific measurements are used for the purpose of the present study.

During the test, one tooth of the driving gear is cracked on the tooth root. As shown in Figure 4, the gear pair is run under load. When they mesh with each other, the teeth of the driving gear are bent backwards, those of the driven gear forwards. The pitch of the driving gear is decreased, while the pitch of the driven gear is increased. The deformation of the pair of teeth in mesh results from Hertzian deformation on the contact area and the cantilever bending of the teeth. If one of the teeth is cracked or heavy damage appears on the tooth root, the resultant deformation will be larger and causes a larger pitch. The following pair of teeth are engaged prematurely and produce an impact of high magnitude. The meshing impact excites free vibrations of the gear-pair system.

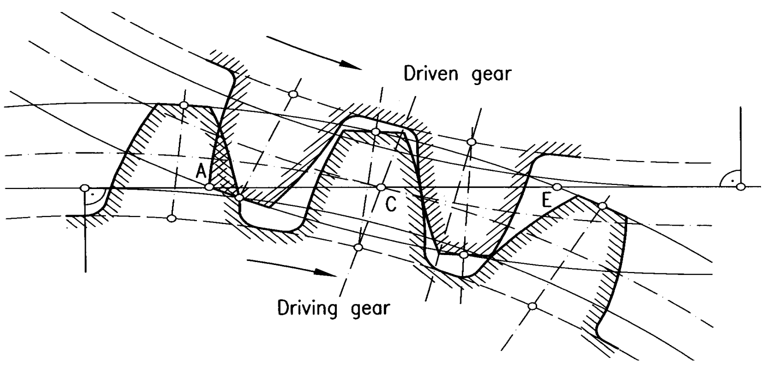

Figure 4. Deformation of the meshing gears

In order to measure these free responses, a Laser Doppler vibrometer has been used for measuring oscillating parts of the angular speed $\dot{\varphi}_1$ and $\dot{\varphi}_2$ of the shafts. The measurement was taken with two non-contacting transducers mounted in proximity to the shafts, positioned at the closest position to the test gears. Figure 5 shows a time record of $\dot{q}(t)$ resulted from this measurement.

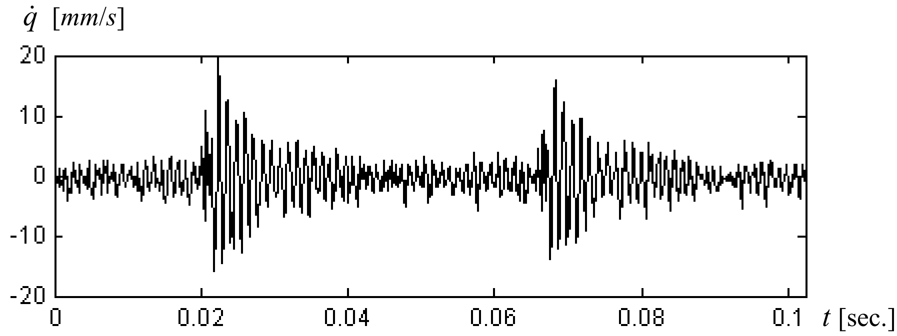

Figure 5. A measured free response of the gear-pair system

Figure 6 displays the wavelet plot of the measured signal. Based on this figure, a wavelet ridge is detected at frequency $f_0 = \omega_0 / 2\pi = 854$ Hz that corresponds to the mean value of the natural frequency $\overline{f_0}$ calculated from the vibration model (see Appendix B). The wavelet envelopes displayed in Figures 7 and 8 are used for estimating the damping ratios. In Figure 9, the results of the damping estimate corresponding to several measurements are given. It can be clearly seen that the values of the damping ratio are very close around the mean value $\overline{\zeta} = 0.024$, and can be used for modelling the gear-pair vibrational system .

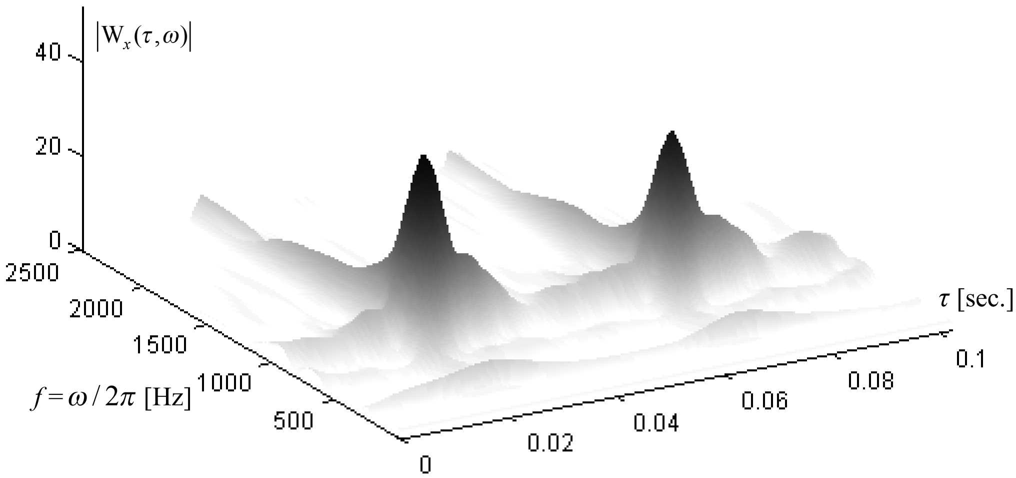

Figure 6. The wavelet plot of the signal shown in Figure 5

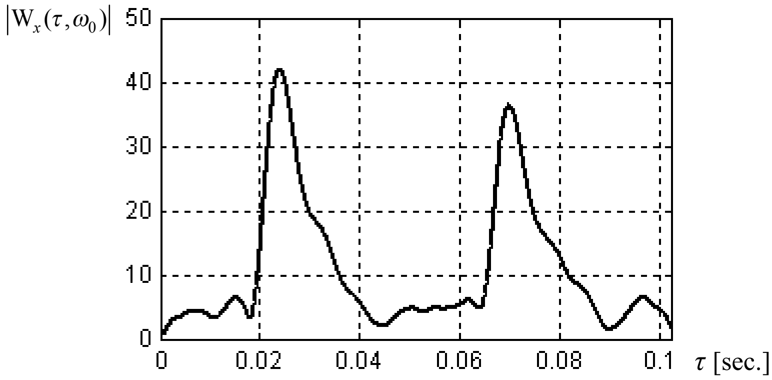

Figure 7. The wavelet envelope extracted from the wavelet plot shown in Figure 6

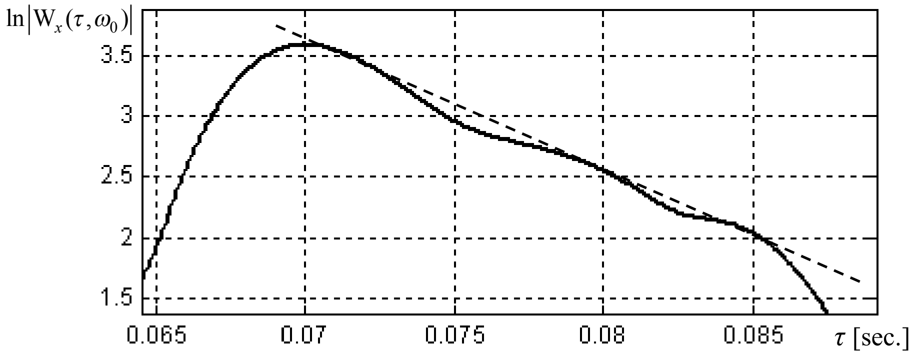

Figure 8. The wavelet envelope (with logarithm scale) for evaluating the damping ratio

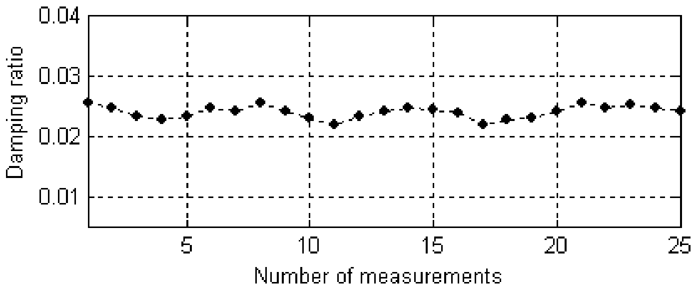

Figure 9. Results of a damping estimate from some measurements on the gearbox

6 Conclusions

A wavelet-based demodulation method for the damping identification from measured free response of linear mdof vibrational systems with viscous damping is presented. The proposed method carries out the CWT of response signals and the extraction wavelet envelopes from wavelet plots in the time-frequency domain. The damping ratio of each vibration mode is then determined by the value of the logarithmic decrement. As can be seen from the identification data, this innovative approach proves to be an effective signal processing tool for the experimental vibration analysis, especially for parameter identification tasks.

First, a simplified mathematical formulation is derived through analytical ways to demonstrate capabilities of the processing method for extracting signal envelopes (Eq. (12)) and evaluating the damping ratios (see Eq. (18)). The numerical simulations have confirmed feasibility of the proposed method.

Second, this study deals with the parameter identification problem in gearboxes by using the proposed demodulation method. The obtained result seems to be very helpful for modeling the gear-pair system, because the inclusion of damping into the analysis significantly expands the usefulness of the analytical calculations. But it should be noted that a complicated gearbox often has complex vibrational response characteristics and numerous natural frequencies of torsion shafts and gear bodies. In this case, the natural frequency of the gear-pair in mesh must be theoretically determined in advance by using vibration models or finite element. This value is required to detect free responses of the meshing gears in the wavelet plot of the measured signal with multi-component as explained.

Appendix A Vibration Model of the Gear-Pair System

The vibration model of the gear-pair system was described in detail in (Nguyen Van Khang et. al, 2004). For the purposes of the present study it is necessary to recall few concepts only.
The differential equations of motion for this system can be expressed in the compact form

$$m_{red}\ddot{q} + d_z\dot{q} + c_z(t)q = F(t) - d_z\dot{e}(t) - c_z(t)e(t) \tag{A1}$$

in which $m_{red} = \dfrac{J_1 J_2}{J_1 r_{b2}^2 + J_2 r_{b1}^2}$, $F(t) = m_{red}\left(\dfrac{M_1(t) r_{b1}}{J_1} + \dfrac{M_2(t) r_{b2}}{J_2}\right)$, $q = r_{b1}\varphi_1 + r_{b2}\varphi_2$

where φ_i ($i = 1,2$) are rotation angle of the input pinion and the output wheel respectively. J_1 and J_2 are the mass moments of inertia of the gears. $M_1(t)$ and $M_2(t)$ denote the external torques load applied on the system. r_{b1} and r_{b2} represent the base radii of the gears, $e(t)$ is a displacement excitation at the mesh. The mesh stiffness $c_z(t)$ is expressed as a time-varying function and the viscous damping coefficient of the gear mesh d_z is assumed to be constant.

The differential equations of motion can be rewritten in the form (Gelman, 2007)

$$\ddot{q} + 2\zeta\omega_0(t)\dot{q} + \omega_0^2(t)q = \frac{F(t)}{m_{red}} - \omega_0^2(t)e(t) - 2\zeta\omega_0(t)\dot{e}(t) \tag{A2}$$

where $\zeta = \dfrac{d_z}{2\sqrt{c_z(t)m_{red}}}$ the damping ratio and $\omega_0(t) = \sqrt{\dfrac{c_z(t)}{m_{red}}}$ the undamped natural frequency.

Appendix B Model Parameters

The following parameters of the model are pre-known: J_1= 9,3.10^{-2} kgm^2; J_2 = 0,272 kgm^2; r_{b1} = 30.46 mm and r_{b2} = 84.66 mm. For this gear-pair, the mesh stiffness $c_z(t)$ is obtained by means of the finite element analysis. In steady state motion of the gear system, the mesh stiffness can be approximately represented by a truncated Fourier series

$$c_z(t) = c_0 + \sum_{k=1}^{K} c_k \cos(k\omega_z t + \gamma_k) \tag{B1}$$

where ω_z is the gear meshing angular frequency which is equal to the number of gear teeth times the shaft angular frequency and K is the number of terms of the series. The values of coefficients are given: c_0 = 8,04.10^8; c_1 = 0,304.10^8; c_2 = 0,185.10^8; c_3 = 0,050.10^8 N/m with corresponding phase angular γ_1 =1,02; γ_2 = -0,72; γ_3 = -0,93 rad. So, the mean value $\bar{\omega}_0$ of the undamped natural frequency can be determined as $\bar{\omega}_0 = \sqrt{\dfrac{c_0}{m_{red}}} \approx 5397\ s^{-1}$, corresponding to $\bar{f}_0 = \bar{\omega}_0/2\pi \approx 859$ Hz.

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