

# Cornering Path of the Vehicle in Case of Sliding

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*When a vehicle is turning at high speed, the side force acting on the tyre produces a side slip angle. If the speed is large enough and the slip angle reaches a maximum value, the tyre begins to slide laterally. In this case the swept path of the vehicle is quite different in comparison with the one in the non-sliding case. This paper presents some research results of dynamics of vehicles cornering at high speed with tyre sliding and also a swept path for this case. A dynamical model is created based on the calculation of the generalized reaction forces of constraints of the mechanical model under consideration. With these reaction forces a computation algorithm is created that enables to include the nonlinear properties of the model as well as of the tyre. Some calculated simulation results are shown for illustration.*

## 1 Introduction

This paper presents a contribution to vehicle dynamics with emphasis on the influence of tyre properties. The nonlinear effects based on shape factors of tyre characteristics on vehicle handling properties will be analysed. The slope of the side force  $F_y$  versus slip angle  $\delta$  is the determining parameter for the handling and stability behavior of automobiles. Especially when the vehicle is cornering at high speed, the side force acting on the tyre is greater than the contact force and tyre sliding phenomena occur. In this paper shape characteristics of the relation between side force and slip angle of the tyre is included in the dynamic model. We use the Principle of Compatibility and numerical methods to solve the obtained equations of motion. The results of the model are compared to the one of a model in which the tyres are non-sliding.

## 2 Tyre model with nonlinear characteristics

The condition for assuring a vehicle being able to move without sliding is that the friction factor  $\varphi$  in the contact area of the tyres and road is large enough in all directions. This condition of the non-sliding vehicle is satisfied when the trailing force  $F_k$  is smaller than limit contact force  $F_{\varphi}$  i.e:

$$F_k \leq F_{\varphi} \quad (1)$$

The limit contact force  $F_{\varphi}$  can be calculated from

$$F_{\varphi} = \varphi \cdot F_z \quad (2)$$

where  $F_z$  is the vertical reaction force from road to the wheel and  $\varphi$  is the friction ratio. For the elastic wheel the situation is more complicated since the friction ratio  $\varphi$  is not constant and is a function of many factors (Wong, 1993). Hence it is reasonable to include all quantities like the deformation of the tyres as well as forces in the dynamical model. From the vehicle dynamical equations these forces can be calculated and the sliding condition could be checked at the same time.

When the vehicle is cornering, side slipping could occur under the affect of side force  $F_y$  which is an effect of centrifugal forces. The direction of the tyre to velocity may be tilted by angle  $\delta$  as shown in Figure 1. There is the relation between contact side force in direction y and angle  $\delta$  and if the side force  $F_y$  is larger than the contact force the wheel will slide laterally (Wong, 1993).

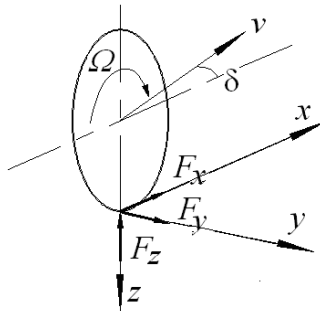


Figure 1. Tyre slip angle and side force

A number of factors affect the cornering behavior of pneumatic tyres such as material of the tyre, friction of the road, the pressure of the tyre, normal load acting on the tyre and others conditions. The normal load on the tyre strongly influences the cornering characteristics. Figure 2 shows the relation of slip angle of the tyre, side force and load at speed 30km/h.

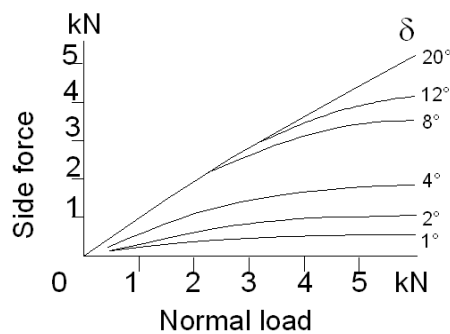


Figure 2. Effect of the normal load on the cornering characteristics of tyres.

The calculation of side force  $F_y$  and checking side slipping is not an easy task since the relation between force and tyre deformation is extremely complicated and can be specified only by experiments on individual tyres or calculated by finite element method (FEM). For including both quantities  $F_y$  and  $\delta$  in the dynamical model, in this article we just deal with the case when the vehicle is running with tyres deformed according to shape characteristics shown in Figure 3 (Pacejka, 2002). The relation is strongly nonlinear and we can divide it into three segments. Segment OA represents the linear deformation of the tyre under the side force with a constant slope coefficient. Segment AB represents nonlinear side slip and B is the point where the tyre slips completely. In the segment BC the tyre is sliding with large deformation and the side force will cause large slip angles.

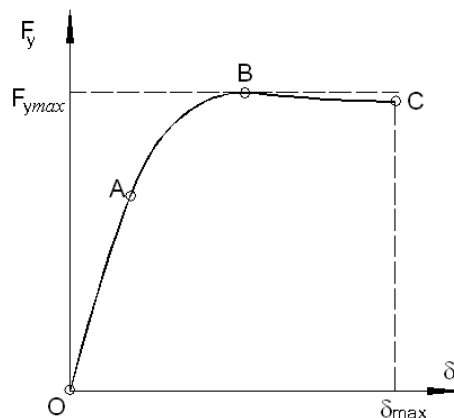


Figure 3. Tyre slip angle characteristics acting by side force.

Based on this relation, we consider two states of motion for the simulation algorithm. In the first state the vehicle runs without sliding but with deformed tyres as presented in segment OB of the shape. In the second state the

required side force is larger than the maximum contact force (at point B) and the wheel is sliding. In this state the slip angle of the wheel can still grow up. So in the mathematical model we have two cases of calculation:

+ Case 1: Slip angle of the wheel is calculated according to the segment OB if the side force increases from 0 to maximum contact force.

+Case 2: If the required side force is beyond the maximum contact force, the wheels are sliding and the maximum value of contact force will be used and the slip angle of the wheel grows to the maximum value.

Both cases should be included in the dynamical equation of model for computer processing. The implicit relation and iterative technique assure the right determination of the slip angle and contact forces.

### 3 Nonlinear dynamic model

Consider the model of the vehicle shown in Figure 4. Parameters used in the vehicle's model are defined as below:

$m$  body mass of vehicle [kg]

$l$  wheelbase of vehicle [m]

$M$  center of gravity

$a$  distance form center of gravity to front axle [m]

$b$  distance form center of gravity to rear axle [m]

$v$  traveling velocity of point B [m/s]

$\theta$  oblique angle of body centerline to  $x$  axis [rad]

$\alpha$  steering angle [rad]

$\delta_1$  slip angle of front wheel [rad]

$\delta_2$  slip angle of rear wheel [rad]

$C$  center of movement

$J$  vehicle's moment of inertial [kgm<sup>2</sup>]

$x, y$  position of point B

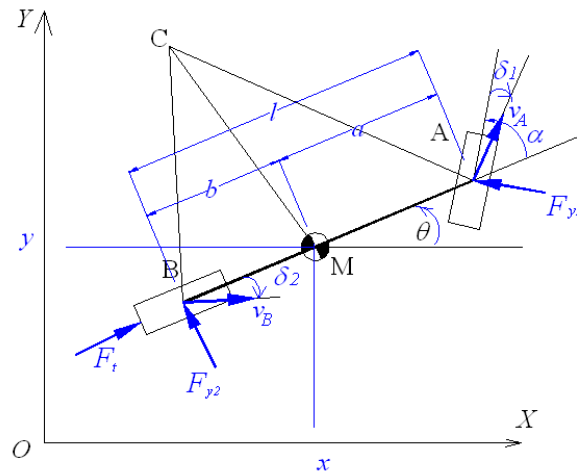


Figure 4. Two-wheel vehicle model during cornering

In this article we use untraditional Principle of Compatibility to solve the problem. The description of the principle can be found in, e.g, Do Sanh et al.(1999). In our case the system of equations of motion is

$$\begin{aligned} m\ddot{x}_M &= F_t \cos \theta + R_x \\ m\ddot{y}_M &= F_t \sin \theta + R_y \\ J\ddot{\theta} &= R_\theta \end{aligned} \quad (3)$$

The generalized reaction forces  $\mathbf{R}$  appear in the equation of motion due to the constraint conditions imposed on the system. They are unknowns in the system of equations and will be determined at each time instant together with generalized coordinates. The relation of these generalized reaction forces and real reaction forces will be

shown further bellow. We will also show that there are two kinematic relations assuring the motion of the wheel in the case under consideration.

The generalized reaction forces  $\mathbf{R} = [R_x \quad R_y \quad R_\theta]^T$  are calculated according to the Principle of Compatibility

$$\mathbf{D}^T \cdot \mathbf{R} = \mathbf{0} \quad (4)$$

where  $\mathbf{D}$  is calculated from the condition

$$\mathbf{G} \cdot \mathbf{D} = \mathbf{0} \quad (5)$$

and  $\mathbf{G}$  is the constraint matrix (Do Sanh et al., 1999).

The constraint equations for matrix  $\mathbf{G}$  follow from the velocity of the center of gravity M.

$$\vec{v}_M = \vec{v}_B + \vec{v}_{M/B} \quad (6)$$

Writing in two scalar components we get

$$\begin{aligned} \dot{x}_M &= \dot{x} - b\dot{\theta} \sin \theta = v \cos(\theta + \delta_2) - b\dot{\theta} \sin \theta \\ \dot{y}_M &= \dot{y} + b\dot{\theta} \cos \theta = v \sin(\theta + \delta_2) + b\dot{\theta} \cos \theta \end{aligned} \quad (7)$$

Eliminating  $v$  gives the constraint equation.

$$\dot{x}_M \sin(\theta + \delta_2) - \dot{y}_M \cos(\theta + \delta_2) + b\dot{\theta} \cos \delta_2 = 0 \quad (8)$$

The other constraint equation is given from the velocity relation between A and M.

$$\begin{aligned} \dot{x}_A &= v_A \cos(\theta + \alpha - \delta_1) \\ \dot{y}_A &= v_A \sin(\theta + \alpha - \delta_1) \end{aligned} \quad (9)$$

Eliminating  $v_A$  we have

$$\dot{y}_A \cos(\theta + \alpha - \delta_1) = \dot{x}_A \sin(\theta + \alpha - \delta_1) \quad (10)$$

From the relation  $\vec{v}_A = \vec{v}_M + \vec{v}_{A/M}$  we have two scalar components.

$$\begin{aligned} \dot{x}_A &= \dot{x}_M - a\dot{\theta} \sin \theta \\ \dot{y}_A &= \dot{y}_M + a\dot{\theta} \cos \theta \end{aligned} \quad (11)$$

Putting (11) into (9) we get

$$\dot{x}_M \sin(\theta + \alpha - \delta_1) - \dot{y}_M \cos(\theta + \alpha - \delta_1) - a\dot{\theta} \cos(\alpha - \delta_1) = 0. \quad (12)$$

Equations (8) and (12) are constraint equations, in the matrix form the constraint matrix  $\mathbf{G}$  is given as

$$\mathbf{G} = \begin{bmatrix} \sin(\theta + \delta_2) & -\cos(\theta + \delta_2) & b \cos \delta_2 \\ \sin(\theta + \alpha - \delta_1) & -\cos(\theta + \alpha - \delta_1) & -a \cos(\alpha - \delta_1) \end{bmatrix}. \quad (13)$$

From this matrix  $\mathbf{G}$  the numerical method can be applied for calculating the matrix  $\mathbf{D}$  and using (4) we can get the generalized reaction forces  $\mathbf{R}$ .

The next task is to calculate the effect of tyre deformation. The relationship between side force and slip angle is discussed in the section 2. We use the shape characteristics to get slip angle of the wheel from the given side force. Formally we can write:

$$\begin{aligned} F_{y1} &= f_1(\delta_1) \\ F_{y2} &= f_2(\delta_2) \end{aligned} \quad (14)$$

where  $f_1$  and  $f_2$  are numerical functions for the front and rear wheel shape characteristics. However in our problem all quantities  $F_{y1}$ ,  $F_{y2}$  and  $\delta_1$ ,  $\delta_2$  are unknowns in the equations of motion. Hence, the iteration should be used. This leads to more accurate results with the nonlinear properties. Of course, the algorithms for the solving the system of equation is more complex. The flow chart is given in Figure 5.

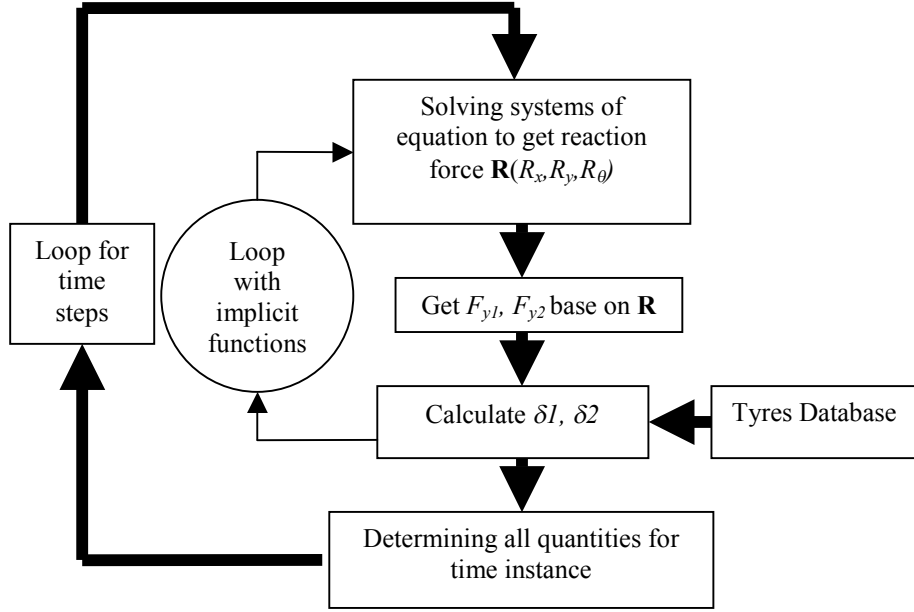


Figure 5. Flow chart of calculation

To determine  $F_{y1}$  and  $F_{y2}$  we use balancing the virtual work of the generalized reaction forces  $\mathbf{R}$  in the directions of generalized coordinates. Therefore in particular case we can determine two components of the force in  $x$  and  $y$  directions as

$$\begin{aligned} -F_{y2} \sin \theta - F_{y1} \sin(\theta + \alpha) &= R_x \\ -F_{y2} \cos \theta - F_{y1} \cos(\theta + \alpha) &= R_y \end{aligned} \quad (15)$$

From (3), (13), (14) and (15) we get a mixed set of differential-algebraic equations. These equations could be solved only by numerical methods for unknowns  $x_M, y_M, \theta, R_x, R_y, R_\theta, F_{y2}, F_{y1}, \delta_1, \delta_2$ .

To make a car model that generates smooth paths, the steering angle is changed as a state variable and the input is the angular velocity  $\omega$  of the steering angle.

$$\dot{\alpha} - \omega = 0 \quad (16)$$

Thus, we have all sets of differential-algebraic equations and can solve this system by using DAESOL package. This package is developed at the Department of Applied Mechanics at Hanoi University of Technology.

#### 4 Simulation results

For illustration we show the results of the model with the proposed simulation algorithm. Parameters for the model are chosen as:

$$\begin{aligned} L &= 3.0 & (\text{m}) \\ a &= 1.6 & (\text{m}) \\ b &= 1.4 & (\text{m}) \end{aligned}$$

$F_t = 0.0$  (N)  
 $J = 200.0$  (Nm<sup>2</sup>)  
 $m = 1600$  (kg)

For simulation some chosen parameters for turning right are:

$\omega = -0.20$  (rad/sec).

Velocities at the beginning of the turning are  $v_0 = 5.0$  m/s and  $v_0 = 7.0$  m/s, respectively.

We calculate the paths of the vehicle for several cases between non-sliding and sliding cases to compare. For this purpose we consider two cases of wheel:

- Rigid wheels with only nonholonomic constraint.
- Elastic wheels with characteristics according to Figure 3.

When using rigid tyres, the result shows that the vehicle will move without sliding. The swept path is depicted in Figure 6.

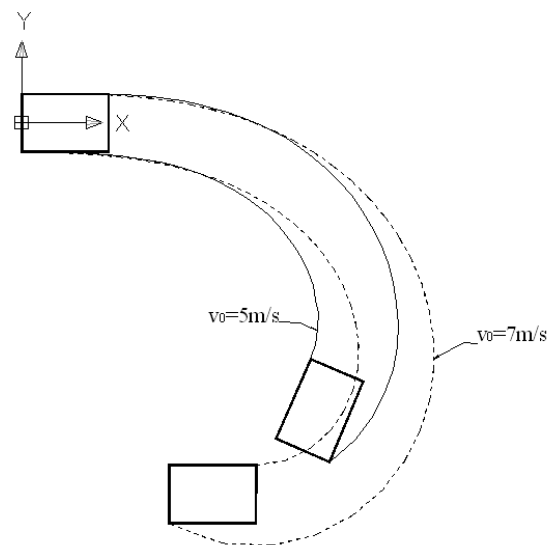


Figure 6. Non-sliding swept path with rigid wheels.

If in the dynamical model we use the tyre with shape characteristic as shown in the Figure 3 the resulting path is shown in the Figure 7.

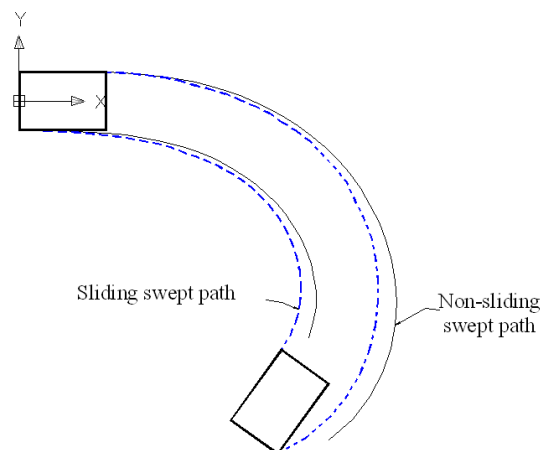


Figure 7. Sliding swept path

In this case we can see that the swept path of the vehicle in sliding (dashed curves) is quite different in comparison with the non-sliding one (solid curves).

Very interesting is also the case where we use the model with large slipping coefficient and tyre is flaccid. The calculated results show instability of the vehicle and it was very hard to determine the path.

## 5 Conclusion

With the proposed model, the dynamics of a turning vehicle in case of sliding tyres under reaction forces shows that tyres have a specific effect on the motion of the vehicle, especially at high velocity. Since real tyres are nonlinear and more complicated, this article considers deformation of tyres under perpendicular forces based on tyre's shape characteristics. The Principle of Compatibility in combination with numerical methods seems to be convenient for determining reaction forces. We can also include in the model the deformation and sliding of the tyre. The comparison of the results from models with and without tyre's deformation and sliding shows that unstable motion of car appears when the tyres are sliding.

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