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Thermal Radiation Effect on Fully Developed Mixed Convection Flow in a Vertical Channel

T. Grosan, I. Pop

The effect of radiation on the steady mixed convection flow in a vertical channel is investigated for laminar and fully developed flow regime. The Rosseland approximation is considered in the modelling of the conduction-radiation heat transfer and temperatures of the walls are assumed constants. The governing equations are expressed in non-dimensional form and are solved both analytically and numerically. It was found that there is a decrease in reversal flow with an increase in the radiation parameters.

1 Introduction

Heat transfer in free and mixed convection in vertical channels occurs in many industrial processes and natural phenomena. It has therefore been the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its applications, for instance, in the design of cooling systems for electronic devices, chemical processing equipment, microelectronic cooling, and in the field of solar energy collection (see Lavine, 1988 and Barletta, 1999). Some of the published papers, such as by Aung (1972), Aung et al. (1972), Aung and Worku (1986a, 1986b), Barletta (2002), and Boulama and Galanis (2004), deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. As is well known, heat exchangers technology involves convective flows in vertical channels. In most cases, these flows imply conditions of uniform heating of a channel, which can be modelled either by uniform wall temperature (UWT) or uniform heat flux (UHF) thermal boundary conditions.

All the above quoted analyses of free and mixed convection flow in vertical channels are based on the hypothesis that the thermal radiation effect within the fluid is negligible. Özişik (1987) has mentioned in an excellent review article that heat transfer by simultaneous radiation and convection has applications in numerous technological problems, including combustion, furnace design, the design of high-temperature gas-cooled nuclear reactors, nuclear-reactor safety, fluidized-bed heat exchangers, fire spreads, advanced energy conservation devices such as open-cycle coal and natural-gas-fired MHD, solar ponds, solar collectors, natural convection in cavities, and many others. On the other hand, it is worth mentioning that heat transfer by simultaneous radiation and convection is very important in the context of space technology and processes involving high temperatures. The inclusion of conduction-radiation effects in the energy equation, however, leads to highly nonlinear partial or ordinary differential equations. The analysis of thermal radiation is complicated by the behaviour of the radiative properties of materials. Properties relevant to conduction and convection (thermal conductivity, kinematic viscosity, density, etc.) are fairly easily measured and are generally well behaved. An excellent description of the fundamentals of thermal radiation has been presented in the book by Modest (2003). For a comprehensive treatment of the radiation transfer and the interactions with conduction and convection the interested readers can consult also the books by Sparrow and Cess (1970), Özişik (1973), and Siegel and Howell (1992).

The aim of the present paper is therefore to analyse the effects of thermal radiation on the steady fully developed mixed convection flow in a vertical channel such that the walls of the channels are subjected to uniform but different wall temperatures (UWT) using the Rosseland approximation model which leads to ordinary differential equations for an optically dense viscous incompressible fluid that flows through the channel. The ordinary differential equations are solved both analytically and numerically using the finite differences method. Flow and heat transfer results for a range of values of the pertinent parameters have been reported. Effects of these parameters, such as the radiation parameter, R_d , the thermal parameter, θ_R , mixed convection parameter

Gr / Re and the entrance temperature parameter, θ_0, on velocity and temperature profiles and on the pressure gradient are shown graphically.

2 Basic Equations

Consider a viscous and incompressible fluid, which steadily flows between two infinite vertical and parallel plane walls. At the entrance of the channel the fluid has an entrance velocity u_0 parallel to the vertical axis of the channel and the fluid temperature is T_0. The distance between the walls, i.e., the channel width is $2L$. A Cartesian coordinate system is chosen such that the $x-$axis is parallel to the gravitational acceleration vector $\mathbf{g}$, but with the opposite direction. The $y-$ axis is orthogonal to the channel walls, and the origin of the axes is such that the positions of the channel walls are $y=-L$ and $y=L$, respectively. A sketch of the system and of the coordinate axes is shown in Figure 1. The wall al $y=-L$ has the given uniform temperature T_1, while the wall at $y=L$ is subjected to a uniform temperature T_2, where $T_2 > T_1$. The fluid velocity vector $\mathbf{v}(u,v)$ is assumed to be parallel to the $x-$ axis, so that the only u does not vanish. The Boussinesq and Rosseland approximations are employed. Fluid rises in the duct driven by

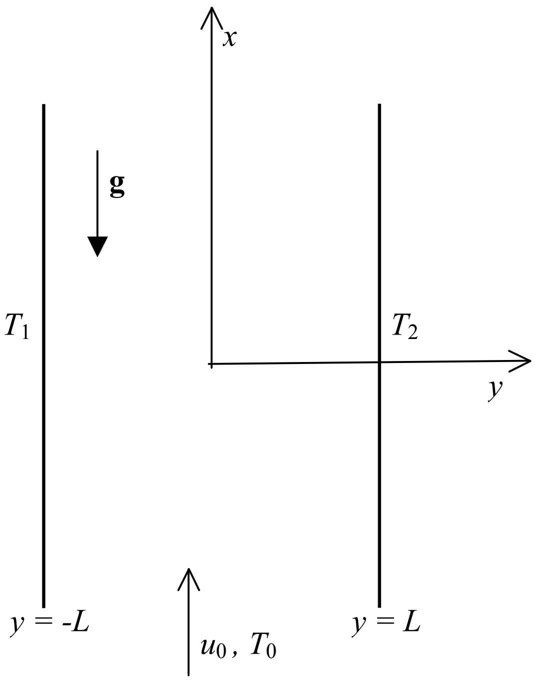


Figure 1. Geometry of the problem and co-ordinate system.

buoyancy forces and initial velocity u_0. Hence, the flow is due to difference in temperature and in the pressure gradient. All the fluid properties except density in the buoyancy term are considered as constant. The flow being fully developed the following relations apply here

$$v=0, \quad \frac{\partial v}{\partial y}=0, \quad \frac{\partial p}{\partial y}=0 \tag{1}$$

where p is the fluid pressure. Therefore, the continuity equation gives $\partial u / \partial x = 0$. One can thus conclude that u does not depend on x, i.e. $u=u(y)$. Under these assumptions the momentum and energy equations for the flow and heat transfer are

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho_0 \, g \, \beta \, (T - T_0) = \frac{dp}{dx} \tag{2}$$

$$k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q^r}{\partial y}=0 \tag{3}$$

where T is the fluid temperature, k is the thermal conductivity of the fluid, β is the thermal expansion coefficient, μ is the dynamic viscosity, ρ_0 is the characteristic density of the fluid, q^r is the radiation heat flux and T_0 is the inlet temperature. We assume that q^r under the Rosseland approximation has the following form, see Modest (2003)

$$q^r=-\left(\frac{4\sigma}{3\chi}\right)\frac{\partial T^4}{\partial y} \tag{4}$$

where σ is the Stefan-Boltzman constant and χ is the mean absorption coefficient. Equations (2) and (3) have to be solved subject to the boundary conditions

$$u(\mp L)=0,\quad T(-L)=T_1,\quad T(+L)=T_2 \tag{5}$$

The closure of the system (2) – (3) subject to the boundary conditions (5) is given by the mass flux conservation equation,

$$u_0=\frac{1}{2L}\int_{-L}^{L}u(y)dy \tag{6}$$

In order to solve equations (2) and (3), we introduce the following non-dimensional variables

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U(Y)=\frac{u}{u_0},\quad \theta(Y)=\frac{T-T_m}{T_2-T_m},\quad P=\frac{pL}{\mu\, u_0} \tag{7}$$

where $T_m=(T_1+T_2)/2$ is the characteristic temperature, see Baytaş et al. (2001). Substituting (7) into equations (2) and (3) we obtain the following non-dimensional ordinary differential equations:

$$\frac{d^2U}{dY^2}+\frac{Gr}{Re}(\theta+\theta_0)=\frac{dP}{dX} \tag{8}$$

$$\frac{d}{dY}\left\{\left[1+\frac{4}{3}R_d\left[1+(\theta_R-1)\theta\right]^3\right]\frac{d\theta}{dY}\right\}=0 \tag{9}$$

where dP/dX in equation (8) should be constant, i.e. $dP/dX=\text{constant}$. Equations (8) and (9) are subject to the boundary conditions (5), which become in dimensionless form

$$U(\mp 1)=0,\quad \theta(-1)=-1,\quad \theta(1)=1 \tag{10}$$

and the conservation mass flux relation (6) takes the form

$$\int_{-1}^{1}U(Y)dY=2 \tag{11}$$

Here Gr is the Grashoff number, Re is the Reynolds number and θ_0 is the entrance temperature parameter defined as

$$Gr=\frac{g\beta(T_2-T_m)L^3}{\nu^2},\quad Re=\frac{u_0L}{\nu},\quad \theta_0=\frac{T_m-T_0}{T_2-T_m} \tag{12}$$

In addition, R_d is the radiation parameter and θ_R is the temperature parameter given by

$$R_d = \frac{4\sigma T_m{}^3}{k\chi}, \quad \theta_R = \frac{T_2}{T_m} \tag{13}$$

We notice that in the case when the radiation effect is absent ($R_d = 0$), equations (8) and (9) are similar to those obtained by Aung and Worku (1986b). However, we did not use the same dimensionless variables as in Aung and Worku (1986b) where it was used T_0 as a reference temperature, while we have used $T_m = (T_1 + T_2)/2$.

The analytical solution of equations (8) – (10) in the case when the radiation effect is absent ($R_d = 0$) can be expressed as

$$U(Y) = -\frac{1}{6}\frac{Gr}{Re}\left(Y^3 - Y\right) - \frac{3}{2}Y^2 + \frac{3}{2}, \quad \theta(Y) = Y, \quad \frac{dP}{dX} = 3 - \frac{Gr}{Re}\theta_0 \tag{14}$$

The physical quantities of interest in this problem are the skin friction coefficient and the Nusselt number which are defined as

$$C_f = \frac{\tau_w}{\rho u_0{}^2}, \quad Nu = \frac{h_w L}{k} \tag{15}$$

where the skin friction τ_w and the convective heat flux coefficient at the walls h_w are given by

$$\tau_w = \mu\left(\frac{du}{dy}\right)_{y=\pm L}, \quad -k\left.\frac{dT}{dy}\right|_{y=\pm L} + \left.q^r\right|_{y=\pm L} = h_w\left[\left.T(y)\right|_{y=\pm L} - T_m\right] \tag{16}$$

Using (7) and (15), we obtain

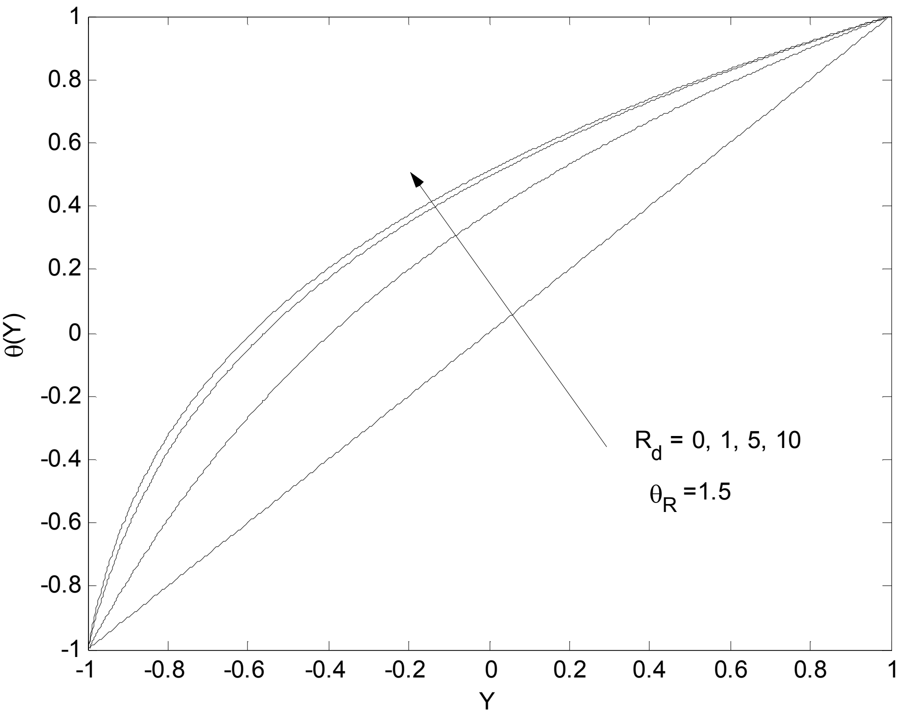
$$C_f\, Re = \left(\frac{dU}{dY}\right)_{Y=\pm 1}, \quad Nu_1 = -\left(1 + \frac{4}{3}R_d\left(2 - \theta_R\right)^3\right)\left(\frac{d\theta}{dY}\right)_{Y=-1}$$
$$Nu_2 = -\left(1 + \frac{4}{3}R_d\theta_R{}^3\right)\left(\frac{d\theta}{dY}\right)_{Y=1} \tag{17}$$

3 Results and Discussion

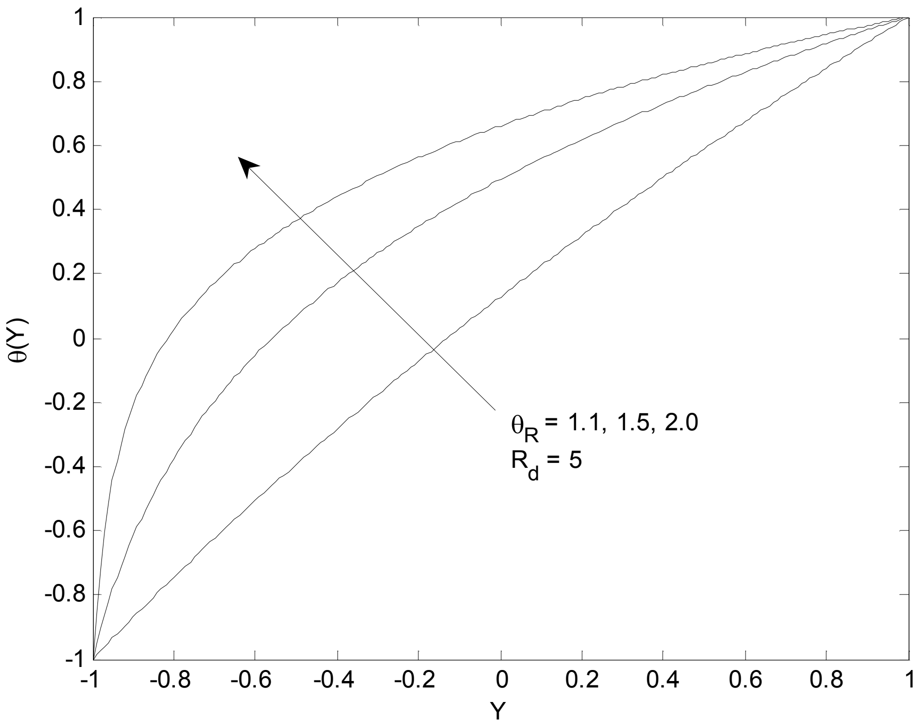
Equations (8) and (9) subject to (10) and (11) were solved numerically for different values of the parameters R_d and θ_R (R_d = 0, 1, 5, 10 and θ_R = 1.1, 1.5, 2.0) using an implicit finite-difference method. It was found that both the Nusselt numbers $|Nu_1|$ and $|Nu_2|$ are equal, so that we present in Table 1 results only for $|Nu_1|$. It is seen that the values of Nu_1 decrease with the increase of the radiation parameter R_d and the temperature parameter θ_R.

Dimensionless temperature profiles $\theta(Y)$ are presented in Figure 2. We notice that the thickness of the temperature profile increase with the increasing of the parameters R_d and θ_R. The effect of the radiation parameter on the velocity profile is presented in Figure 3. In Figure 3a the analytical solution given by equation (14) is also included and the agreement with the numerical results is very good. Therefore, we are very confident with the present results. The increases of both radiation and temperature parameters diminish the reversal flow, thus the radiation effect is to stabilize the fluid motion as can be seen from Figure 3.

Figure 4 illustrates the influence of the mixed convection parameter, Gr/Re, on the dimensionless velocity profile, $U(Y)$. We notice an increase of the reversal flow with the increasing of the mixed convection parameter (see Figure 4a) and that the reversal flow is absent for small values of the mixed convection parameter (see Figure 4b) In order to study the influence of the radiation parameter R_d on the reversal flow, we have determined the variation of

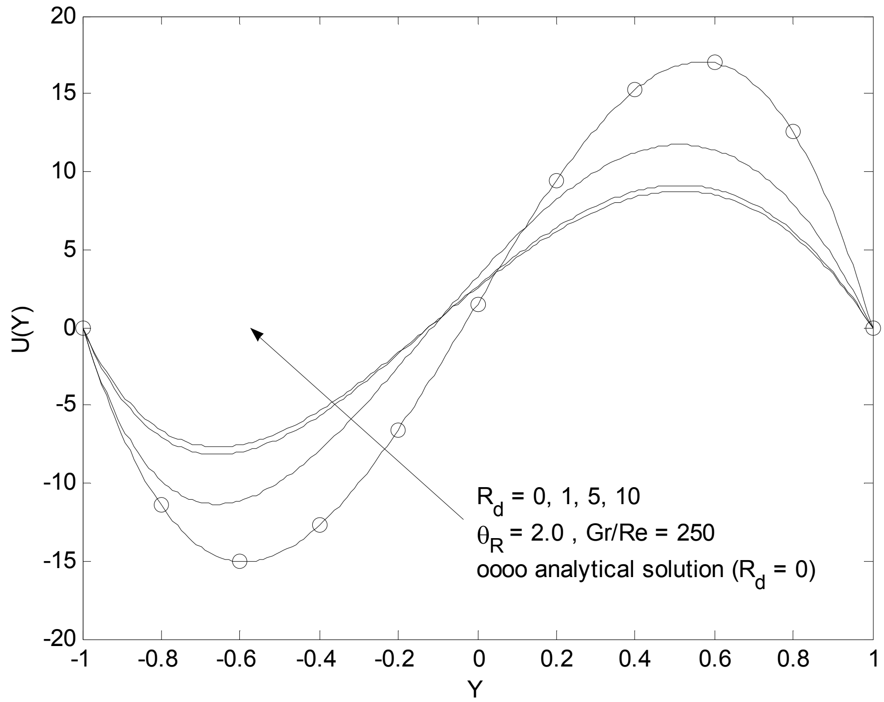


a)



b)

Figure 2. Variation of dimensionless temperature profiles for a) different values of the radiation parameter R_d and b) different values of the temperature parameter θ_R .



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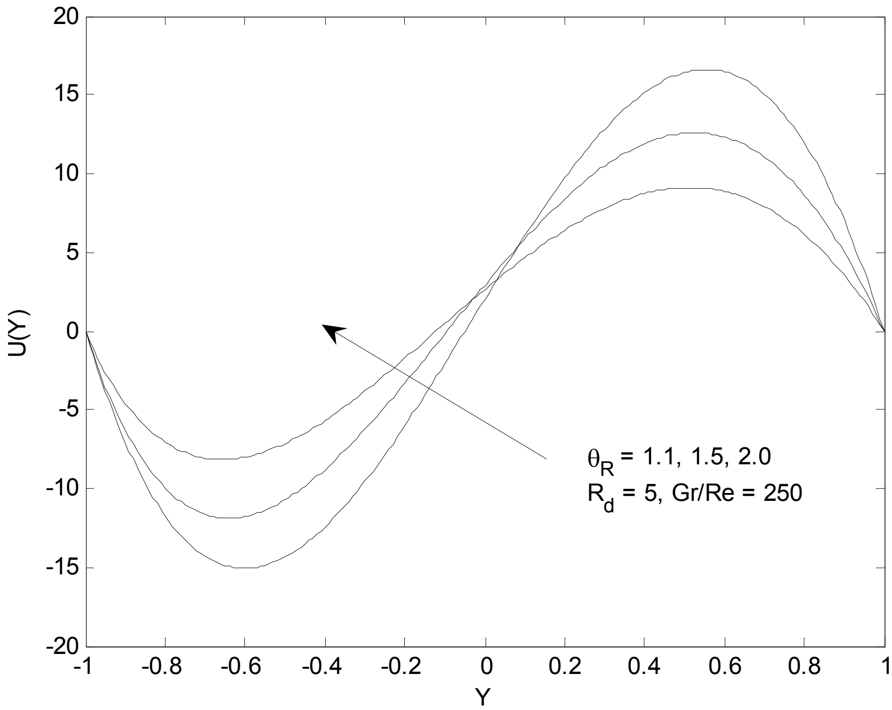

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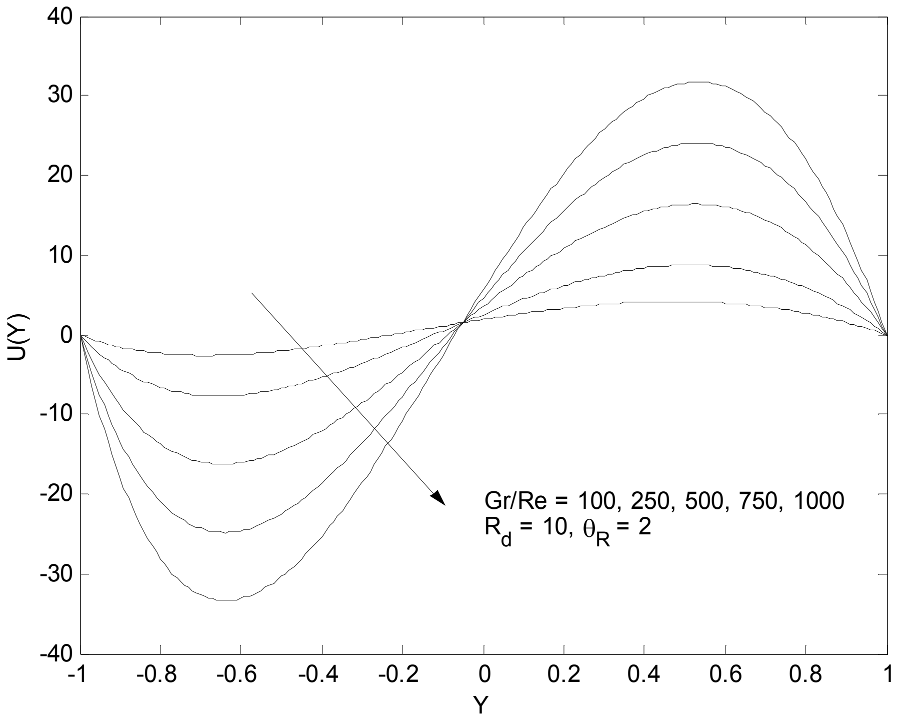
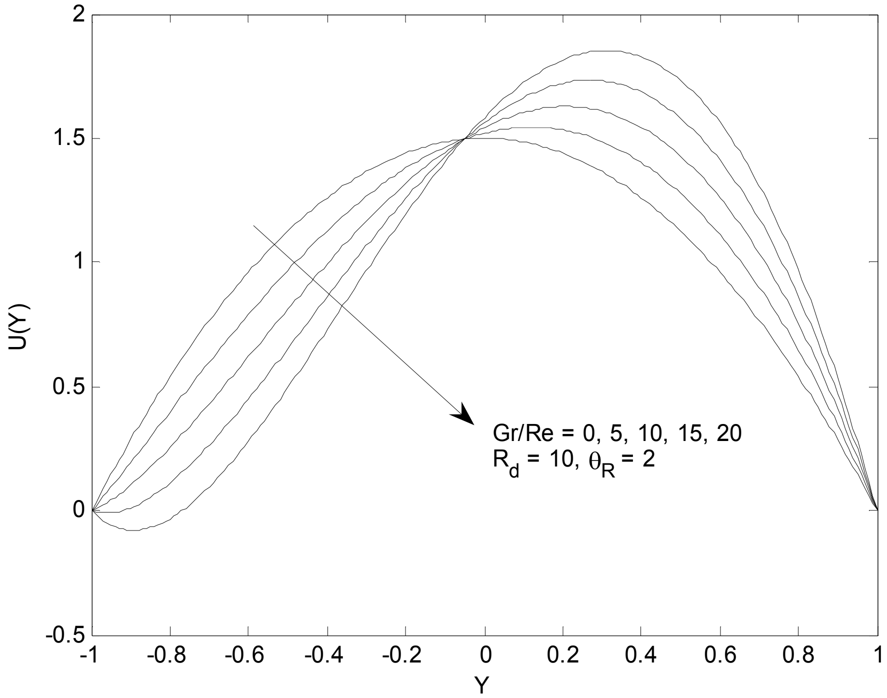
Figure 3. Variation of dimensionless velocity profiles for a) different values of the radiation parameter R_d and b) different values of the temperature parameter θ_R .



a)



b)

Figure 4. Variation of dimensionless velocity profiles for different values of the parameter Gr / Re : a) large values and b) small values.

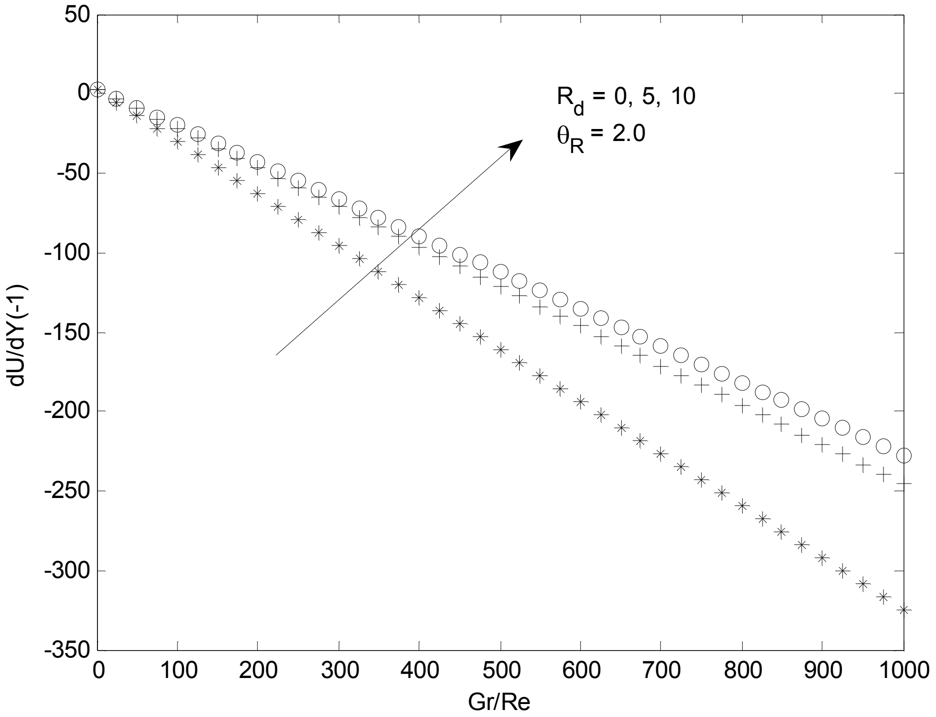
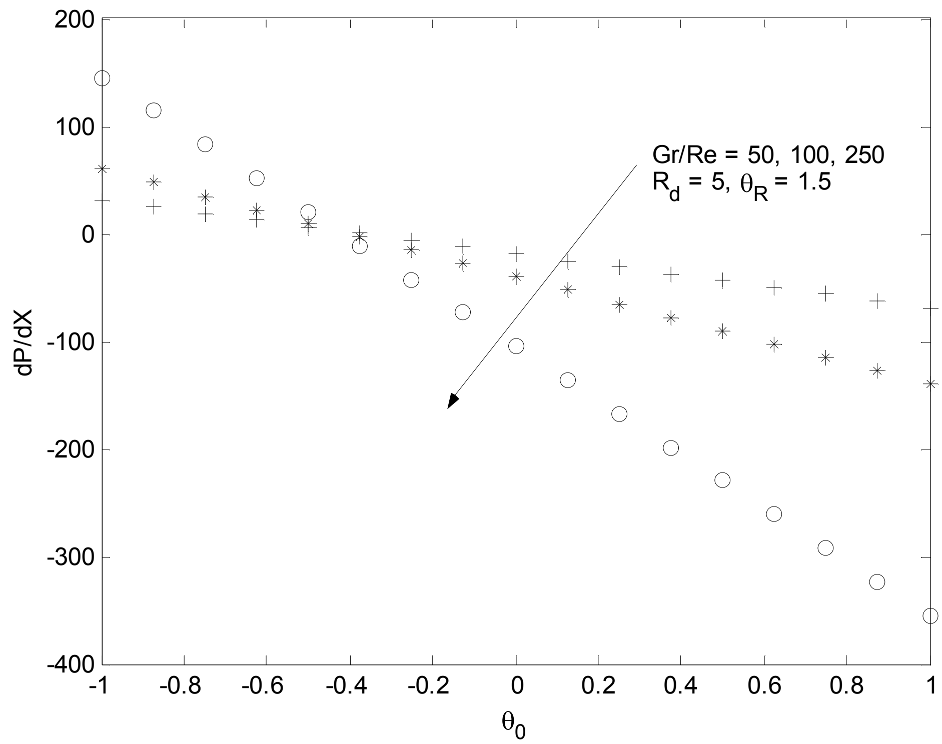
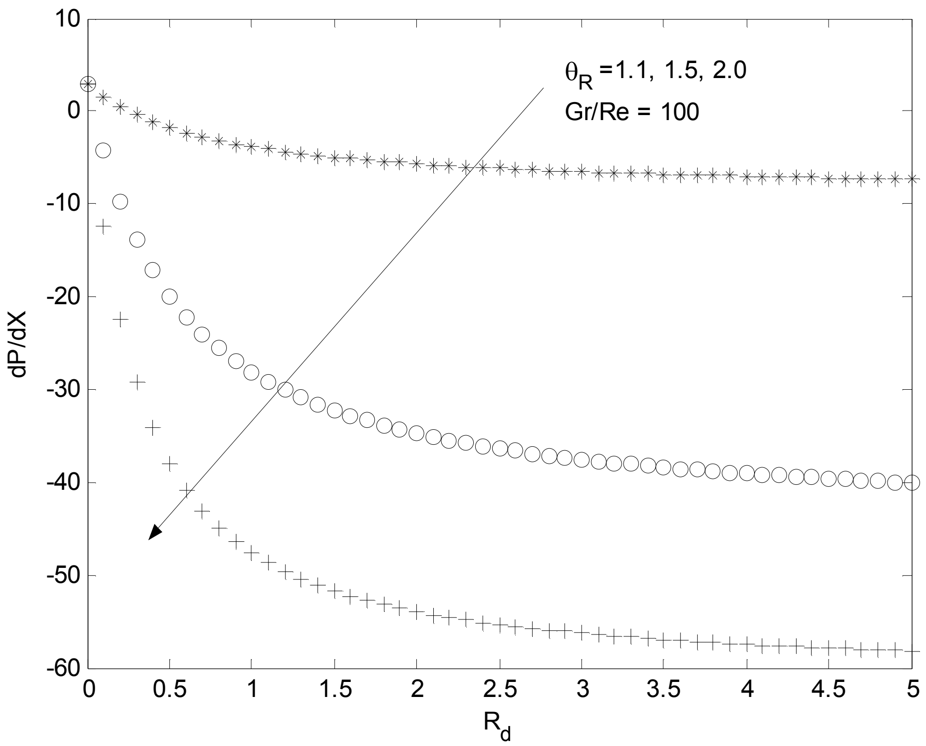


Figure 5. The effect of the radiation parameter R_d on the flow reversal.



a)



b)

Figure 6. Variation of the pressure gradient with a) the temperature parameter θ_0 and b) the radiation parameter R_d.

the velocity gradient, $(dU/dY)_{y=-1}$, which is related to the skin friction, for some values of the radiation parameter R_d. We conclude that the influence of R_d on the reversal flow is more sensitive for large values of the mixed convection parameter Gr/Re (see Figure 5). Values of the skin friction coefficient, $C_f Re$, on the both walls are given in Table 2 for some values of the parameters Gr/Re, R_d and θ_R. We notice that $C_f Re$ decrease when Gr/Re increases for all values of R_d and θ_R considered. However, for large values of the parameters R_d and θ_R, the skin friction coefficient $C_f Re$ increases.

R_d	θ_R	$\lvert Nu_1 \rvert$
1	1.1	2.346
	1.5	2.666
	2.0	3.667
5	1.1	7.733
	1.5	9.318
	2.0	14.334
10	1.1	14.465
	1.5	17.613
	2.0	27.668

Table 1. Values of the Nusselt number for different values of the parameters R_d and θ_R.

The variation of the pressure gradient dP/dX with the parameters θ_0 and R_d is shown in Figure 6. It is seen that dP/dX decreases with the increasing of the temperature parameter θ_0, the effect being more sensitive for large values of Gr/Re (see Figure 6a). On the other hand, Figure. 6b illustrates the variation of dP/dX in respect with R_d. It is seen that dP/dX decreases with the increasing of R_d, the effect being more present for large values of θ_R.

Gr / Re	R_d	θ_R	$\left(\frac{dU}{dY}\right)_{Y=-1}$	$\left(\frac{dU}{dY}\right)_{Y=1}$	Gr / Re	R_d	θ_R	$\left(\frac{dU}{dY}\right)_{Y=-1}$	$\left(\frac{dU}{dY}\right)_{Y=1}$
10	1	1.1	-0.43109	-6.20181	100	1	1.1	-31.29746	-35.03162
		1.5	-0.56558	-5.58934			1.5	-32.64237	-28.90696
		2.0	-0.33724	-4.93834			2.0	-30.35892	-22.39694
	10	1.1	-0.47180	-6.10705		10	1.1	-31.70459	-34.08400
		1.5	-0.17617	-5.08102			1.5	-28.74825	-23.82377
		2.0	0.61450	-4.42731			2.0	-20.84146	-17.28663
500	1	1.1	-168.48134	-163.16414	1000	1	1.1	-339.96117	-323.32978
		1.5	-175.20589	-132.54083			1.5	-353.41029	-262.08315
		2.0	-163.78863	-99.99071			2.0	-330.57576	-196.98292
	10	1.1	-170.51699	-158.42600		10	1.1	-344.03247	-313.85349
		1.5	-155.73525	-107.12486			1.5	-314.46901	-211.25121
		2.0	-116.20134	-74.439163			2.0	-235.40117	-145.87982

Table 2. Values of the skin friction coefficient $C_f Re$ for different values of the parameters Gr / Re, R_d and θ_R.

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Addresses: Dr. T. Grosan,, Faculty of Mathematics and Computer Science, Babeş-Bolyai University Cluj-Napoca, Romania; Professor I. Pop, Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania.