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Generalizations of Maysel's Formula to Micropolar Thermoviscoelasticity with non-small Temperature Changes

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Generalizations of Maysel's formula to micropolar thermoviscoelasticity are given. The coupled term in generalized thermoelasticity formulation is modified with non-small temperature changes, where the absolute temperature is not replaced by the temperature of the body in its undeformed state, and is expressed as a linear function of time. The term including $((d/ds)\bar{u}_{i,i})$ in the Laplace transform domain is treated with an approximate method. The new reciprocity theorem and fundamental solutions of the linear micropolar thermoviscoelasticity with non-small temperature changes in the Laplace transform domain are also derived. To illustrate Maysel's method, a mixed boundary value problem is considered as an example.

1 Introduction

Generalized continuum theories for mechanical behavior developed over the last century admitted degrees of freedom which were not considered in the classical theory of elasticity. The micropolar elasticity theory takes into consideration the granular character of the medium, and is intended to be applied to materials for which the ordinary classical theory of elasticity fails owing to the microstructure of the material. Within such a theory, solids can undergo macro-deformations and micro-rotations. The motion in this kind of solids is completely characterized by the displacement vector $\mathbf{u}(\mathbf{x}, t)$ and the rotation vector $\phi(\mathbf{x}, t)$, while in the case of classical elasticity, the motion is characterized by the displacement vector only. The general theory of linear micropolar thermoelasticity was given by Eringen (1970) and Nowacki (1974).

Since the works of Maxwell, Boltzmann, Voigt, Kelvin, and others, the linear viscoelasticity has remained an important area of research. Gross (1953) investigated the representations of mechanical models with linear viscoelastic behavior.

In recent years the micropolar thermoviscoelasticity has gained great importance due to the large-scale development and utilization of composite, reinforced, and coarse-grained materials. The micropolar viscoelasticity theory has been investigated by many authors (see Eringen (1967)).

Biot (1965) formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory, namely that elastic changes have no effect on the temperature. The heat equations for both theories are of parabolic type predicting infinite speeds of propagation for heat waves contrary to physical observations. To eliminate this paradox, many generalized thermoelasticity theories have been developed subsequently. Hetnarski and Ignaczak (1999) in their survey article examined five generalizations of the coupled theory. Consequently, the dynamic problem of generalized thermoelasticity must be formulated with mutual coupling of both the heat conduction equation and the equation of motion. The effect of coupling is not small for some synthetic materials. However, since the absolute temperature T in the coupled term $T\dot{u}_{i,i}$ is not constant, the governing equations are nonlinear. Atkinson (1991) derived the coupled thermoelasticity equations with a temperature term that does not depend on $(T - T_0)/T$ (T_0 is the temperature of body in its undeformed state) being small, and derived several solutions limited to quite specialized loads and boundary conditions. We must point out that, however, only when $(T - T_0)/T \ll 1$, the assumption of small temperature changes does not lead to a large error. In the situation with large temperature changes, the problem is too complicated to solve (Zhong and Zhang, 2001).

The technique frequently used in isothermal elasticity, known as Betti's method, has been extended to thermoelasticity by Maysel (1951), who deduced a method of integration of the boundary value problem of thermoelasticity. The Maysel formula to determine the displacement $u_j(\mathbf{x})$ in a body D, due to the action of a steady temperature field $T(\mathbf{x})$, has the form $u_j(\mathbf{x}) = a\int_D T(\mathbf{y})\sigma_{kk}^{(j)}((\mathbf{y},\mathbf{x}))dV(\mathbf{y})$, where $\sigma_{kk}^{(j)}$ is the sum of normal stresses at the point $\mathbf{y}$ of the elastic body in the isothermal state ($T = 0$), due to the action of a concentrated unit force located at the point $\mathbf{x}$ in the direction of the x_j-axis.

The paper is devoted to a generalization of Maysel's formula to a theory of thermovicoelasticity with non-small temperature changes. In order to solve this kind of problems, several simplifying assumptions applicable to usual applications are adopted in this paper. The topic of the numerical resolution will be treated by the author in a future paper. One can refer to Ziegler and Irschik (1987, 1985) and Ziegler (2004) for the methods of solution in thermoleasticity based on Maysel's formula and its implementation in the direct boundary integral methods.

Nomenclature

u_i	components of displacement vector
σ_{ij}	components of force stress tensor
m_{ij}	components of couple stress tensor
e_{ij}	components of strain tensor
ϵ_{ij}	components of micro-strain tensor
δ_{ij}	Kronecker delta
ε_{ijk}	permutation tensor
e	$= e_{kk} = \epsilon_{kk}$ dilatation
r_i	components of rotation vector
ϕ_i	components of micro-rotation vector ($\phi = \phi_{ii} = \phi_{i,i}$)
M_i	mass couple vector
F_i	mass force vector
ρ	density
j	micro-inertia coefficient
a	coefficient of linear thermal expansion
$\lambda,$	$\mu,\ k,\ \alpha,\ \beta,\ \gamma$ elastic coefficients
T	absolute temperature
T_0	reference temperature
θ	$= T - T_0$
K	thermal conductivity
Q	intensity of applied heat source per unit mass
c_E	specific heat at constant strain

2 The Mathematical Problem

Assume that a linear micropolar thermoviscoelastic material occupies a regular region D with a smooth boundary surface B in the three-dimensional Euclidian space. The material is assumed to be microisotropic and isotropic. Through this paper a rectangular coordinate system (x_1, x_2, x_3) is employed. $\mathbf{x}$ is the position vector and t the time. All the functions are considered to be functions of $(\mathbf{x}, t)$ defined on $\bar{D}(= D \cup B) \times [0, \infty)$. A superposed dot denotes differentiation with respect to time, while a comma denotes partial differentiation with respect to the space variables x_i. The summation notation is used. Following Eringen (1967), the system of governing equations of a linear micropolar thermoviscoelasticity consists of

(i) Equations of motion (on $D \times [0, \infty)$)

$$\sigma_{ji,j} + \rho F_i = \rho \ddot{u}_i, \qquad \varepsilon_{ijp}\sigma_{jp} + m_{ji,j} + \rho M_i = j\rho\ddot{\phi}_i \tag{1}$$

(ii) Kinematic relations (on $D \times [0, \infty)$)

$$\epsilon_{ij} = e_{ij} - \varepsilon_{ijp}(r_p - \phi_p), \quad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad r_i = \frac{1}{2}\varepsilon_{ipq}u_{q,p}, \quad \phi = \phi_{i,i} \tag{2}$$

(iii) Constitutive laws (on $\bar{D} \times [0, \infty)$)

$$\sigma_{ij} = \lambda\breve{R}_\lambda(e)\delta_{ij} + (2\mu\breve{R}_\mu + k\breve{R}_k)(e_{ij}) + k\breve{R}_k(\varepsilon_{ijp}(r_p - \phi_p)) - a(3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(\theta)\delta_{ij} \tag{3}$$

$$m_{ij} = \alpha\breve{R}_\alpha(\phi)\delta_{ij} + \beta\breve{R}_\beta(\phi_{i,j}) + \gamma\breve{R}_\gamma(\phi_{j,i}) \tag{4}$$

where the operator $\breve{R}_\xi(f)$, $(\xi = \lambda, \mu, k, \alpha, \beta, \gamma)$ is defined for any function $f(\mathbf{x}, t)$ of class C^1, as

$$\breve{R}_\xi(f) = \breve{R}_\xi(f(\mathbf{x}, t)) = \int_0^t R_\xi(t - \tau)\frac{\partial f(\mathbf{x}, \tau)}{\partial \tau}d\tau$$

where $R_\xi(t)$ are six relaxation functions.

Using the kinematic relations, equation (3) takes the form

$$\sigma_{ji} = \lambda\breve{R}_\lambda(u_{p,p})\delta_{ij} + (\mu\breve{R}_\mu + k\breve{R}_k)(u_{i,j}) + \mu\breve{R}_\mu(u_{j,i}) + k\breve{R}_k(\varepsilon_{ijp}\phi_p) - a(3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(\theta)\delta_{ij} \tag{5}$$

Substituting the last two equations into the equations of motion (1) we get

$$(\lambda\breve{R}_\lambda + \mu\breve{R}_\mu)(u_{j,ji}) + (\mu\breve{R}_\mu + k\breve{R}_k)(u_{i,jj}) + k\breve{R}_k(\varepsilon_{ijp}\phi_{p,j}) - a(3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(\theta_{,i}) = \rho(\ddot{u}_i - F_i) \tag{6}$$

$$(\alpha\breve{R}_\alpha + \beta\breve{R}_\beta)(\phi_{j,ji}) + \gamma\breve{R}_\gamma(\phi_{i,jj}) + k\breve{R}_k(\varepsilon_{ijp}u_{p,j}) - 2k\breve{R}_k(\phi_i) = \rho(j\ddot{\phi}_i - M_i) \tag{7}$$

(iv) The equation of heat conduction has the form (on $D \times [0, \infty)$)

$$K\theta_{,ii} = \rho c_E\dot{\theta} + Ta(3\lambda\breve{R}_\lambda + 2\mu\breve{R}_\mu + k\breve{R}_k)(\dot{u}_{i,i}) - Q \tag{8}$$

The $T = T(\mathbf{x}, t)$ in the coupling term in equation (8) and the heat conduction equation is non-linear. In order to get a closed solution in generalized thermoelasticity (Fung, 1968), $T = T_0$ was taken depending on the assumption $(T - T_0)/T \ll 1$. Now in the situation of non-small temperature changes, we suppose $T = T_0 + Nt$, where N is suitably selected to stimulate the rate of the temperature change, i.e., temperature change is linear with respect to time, then we are able to solve the coupled and non-linear equations (6)-(8).

These equations are the field equations (on $D \times (0, \infty)$) of linear micropolar thermoviscoelasticity, applicable to several special cases as follows

1. The equations of linear micropolar thermoviscoelasticity of a Kelvin-Voigt model can be obtained from the above equations by replacing the operator $\breve{R}_\xi(f)$ by

$$\breve{R}_\xi^{(v)}(f) = (1 + \lambda_v\frac{\partial}{\partial t})f(\mathbf{x}, t)$$

where $\lambda_v > 0$ is the retardation period of the Kelvin-Voigt model (Alfrey and Gurnee, 1956).

2. The equations of linear micropolar thermoelasticity can be obtained from equations (6)-(8) by replacing the operator $\breve{R}_\xi(f)$ by the function $f(\mathbf{x}, t)$.

3. The equations of linear thermoviscoelasticity can be obtained from equations (6)-(8) by setting $k = 0,\ \phi_i = 0$, and $M_i = 0$.

4. The equations of linear thermoelasticity can be obtained from equations (6)-(8) by replacing the operator $\breve{R}_\xi(f)$ by f, and setting $k = 0,\ \phi_i = 0$, and $M_i = 0$.

The system of equations (6)-(8) is completed by the initial and boundary conditions. The initial conditions will be assumed homogeneous

$$u_i(\mathbf{x},t)=0;\ \phi_i(\mathbf{x},t)=0;\ \theta(\mathbf{x},t)=0;\ \mathbf{x}\in\bar{D};\ t\leq 0 \tag{9}$$

$$\frac{\partial^n u_i(\mathbf{x},t)}{\partial t^n}=0,\ \frac{\partial^n \phi_i(\mathbf{x},t)}{\partial t^n}=0,\ \frac{\partial^n \theta(\mathbf{x},t)}{\partial t^n}=0;\ \mathbf{x}\in\bar{D};\ t\leq 0 \tag{10}$$

The boundary conditions

$$\sigma_{ji}n_j=f_i(\mathbf{x},t)\ on\ B_\sigma\times(0,\infty),\ u_i=g_i(\mathbf{x},t)\ on\ B_u\times(0,\infty) \tag{11}$$

$$m_{ji}n_j=\Omega_i(\mathbf{x},t)\ on\ B_m\times(0,\infty),\ \phi_i=\Xi_i(\mathbf{x},t)\ on\ B_\phi\times(0,\infty) \tag{12}$$

$$\theta=\Phi(\mathbf{x},t)\ on\ B_1\times(0,\infty),\ \theta_{,n}=G(\mathbf{x},t)\ on\ B_2\times(0,\infty) \tag{13}$$

where the functions $f_i,\ g_i,\ \Omega_i,\ \Xi_i,\ \Phi$, and G are equal to zero when $t\leq 0$. $(B_u;B_\sigma)$, $(B_1;B_2)$ and $(B_\phi;B_m)$ are three partitions of the boundary surface B such that $B=B_u\cup B_\sigma=B_\phi\cup B_m=B_1\cup B_2$, $B_u\cap B_\sigma=B_\phi\cap B_m=B_1\cap B_2=\emptyset$ and $n_i=n_i(\mathbf{x})$ are the components of the outward-pointing normal vector on the surface at $\mathbf{x}\in B$.

3 The Formulation of the Problem in the Laplace Transform Domain

Performing the Laplace transform defined as

$$\bar{f}(\mathbf{x},s)=\int_0^\infty f(\mathbf{x},t)e^{-st}dt \tag{14}$$

over equations (1), (4) and (5), with homogeneous initial conditions, we obtain

$$\bar{\sigma}_{ji,j}+\rho\bar{F}_i=\rho s^2\bar{u}_i,\qquad \varepsilon_{ijp}\bar{\sigma}_{jp}+\bar{m}_{ji,j}+\rho\bar{M}_i=j\rho s^2\bar{\phi}_i \tag{15}$$

$$\bar{\sigma}_{ji}=\lambda_1\bar{u}_{p,p}\delta_{ij}+(\mu_1+k_1)\bar{u}_{i,j}+\mu_1\bar{u}_{j,i}+k_1\varepsilon_{ijp}\bar{\phi}_p-a(3\lambda_1+2\mu_1+k_1)\bar{\theta}\delta_{ij} \tag{16}$$

$$\bar{m}_{ij}=\alpha_1\bar{\phi}\delta_{ij}+\beta_1\bar{\phi}_{i,j}+\gamma_1\bar{\phi}_{j,i} \tag{17}$$

The field equations (6)-(8) in the Laplace transform domain take the form

$$\begin{aligned}(\lambda_1+\mu_1)\bar{u}_{j,ji}+(\mu_1+k_1)\bar{u}_{i,jj}+k_1\varepsilon_{ijp}\bar{\phi}_{p,j}-a(3\lambda_1+2\mu_1+k_1)\bar{\theta}_{,i}&=\rho(s^2\bar{u}_i-\bar{F}_i), &(18)\\ (\alpha_1+\beta_1)\bar{\phi}_{j,ji}+\gamma_1\bar{\phi}_{i,jj}+k_1\varepsilon_{ijp}\bar{u}_{p,j}-2k_1\bar{\phi}_i&=\rho(js^2\bar{\phi}_i-\bar{M}_i), &(19)\\ \rho c_E s\bar{\theta}+a(3\lambda_1+2\mu_1+k_1)(T_0s-N-Ns\frac{d}{ds})\bar{u}_{i,i}-\bar{Q}&=K\bar{\theta}_{,ii} &(20)\end{aligned}$$

where

$$\xi_1=s\xi R_\xi(s),\ \ \xi=\lambda,\mu,k,\alpha,\beta,\gamma$$

$R_\xi(s)$ is the Laplace transform of the relaxation functions $R_\xi(t)$. ($\xi_1=\xi(1+\lambda_v s)$) for the Kelvin-Voigt model and $\xi_1=\xi$ for the generalized linear micropolar thermoelasticity).

Equations (20) form a set of equations with variable coefficient containing $((d/ds)\bar{u}_{i,i})$ which represents the strain $\bar{e}_{ii}$ differentiation with respect to the transformation parameter s, and this term is difficult to treat numerically. The further study indicates that the term containing $((d/ds)\bar{u}_{i,i})$ can be simplified after a suitable mathematical treatment. $u_i(\mathbf{x},t)$ is supposed to be an odd function with respect to time. Then using a Fourier expansion of the function $u_i(\mathbf{x},t)$, i.e., $u_i(\mathbf{x},t)=\sum_{n=1}^\infty A_{in}(\mathbf{x})\sin n\omega t$, the first-order derivative of which with respect to the coordinate is $u_{i,i}(\mathbf{x},t)=\sum_{n=1}^\infty A_{in,i}(\mathbf{x})\sin n\omega t$, we obtain

$$\bar{u}_{i,i}=\int_0^\infty u_{i,i}e^{-st}dt=\sum_{n=1}^\infty A_{in,i}(\mathbf{x})\frac{n\omega}{s^2+n^2\omega^2},\ \text{ and }\ \frac{d}{ds}\bar{u}_{i,i}=-\sum_{n=1}^\infty A_{in,i}(\mathbf{x})\frac{2n\omega s}{(s^2+n^2\omega^2)^2}$$

Hence

$$\frac{d}{ds}\bar{u}_{i,i} = -\sum_{n=1}^{\infty} \frac{2s}{s^2+n^2\omega^2}\bar{u}_{i,i}$$

So, equation (20) may be written as

$$K\bar{\theta}_{,ii} = \rho c_E s\bar{\theta} + a(3\lambda_1 + 2\mu_1 + k_1)(T_0 s + Nf(s,\omega))\bar{u}_{i,i} - \bar{Q} \tag{21}$$

where $f(s,\omega) = -1 + \sum_{n=1}^{\infty} \frac{2s^2}{s^2+n^2\omega^2}$. Now as long as we select a suitable sine frequency ω according to specific boundary conditions, we can simulate the law of the displacement u_i change with time well.

In the Laplace transform domain, the boundary conditions (11)-(13) are

$$\bar{\sigma}_{ji}(\mathbf{x},s)n_j(\mathbf{x},s) = \bar{f}_i,\ \mathbf{x}\in B_\sigma,\ \bar{u}_i(\mathbf{x},s) = \bar{g}_i,\ \mathbf{x}\in B_u,\ \bar{m}_{ji}n_j(\mathbf{x},s) = \bar{\Omega}_i,\ \mathbf{x}\in B_m \tag{22}$$

$$\bar{\phi}_i(\mathbf{x},s) = \bar{\Xi}_i,\ \mathbf{x}\in B_\phi,\ \bar{\theta}(\mathbf{x},s) = \bar{\Phi},\ \mathbf{x}\in B_1,\ \theta_{,n}(\mathbf{x},s) = \bar{G},\ \mathbf{x}\in B_2 \tag{23}$$

and the homogeneous initial conditions (9)-(10) are given by

$$\bar{u}_i(\mathbf{x},s) = 0,\ \bar{\phi}_i(\mathbf{x},s) = 0,\ \theta(\mathbf{x},s) = 0,\ \mathbf{x}\in\bar{D} \tag{24}$$

4 Reciprocity Theorem

We assume the system of equations (18)-(20) to be given with the boundary conditions (22)-(23) and the homogeneous initial conditions (24). Consider two problems where applied mass forces, mass couples, heat sources, surface tractions, surface couple-stresses, assigned surface displacements, assigned surface microrotations, surface temperature, and the normal derivative $\theta_{,n}$ on the surface are specified differently under zero initial conditions. The action starts at $t = 0^+$ and produces in the body displacement microrotations ϕ_i and a temperature increment θ. Let the variables involved in these two problems be distinguished by superscripts in parentheses. Thus, we have $u_i^{(1)},\ \phi_i^{(1)},\ m_{ij}^{(1)},\ e^{(1)},\ \sigma_{ij}^{(1)},\ \theta^{(1)},\cdots$ for the first problem and $u_i^{(2)},\ \phi_i^{(2)},\ m_{ij}^{(2)},\ e^{(2)},\ \sigma_{ij}^{(2)},\ \theta^{(2)},\cdots$ for the second problem. Each set of variables satisfies the system of equations (18)-(24).

Using the divergence theorem and equations $(15)_1$, (22)-(23), we get (omitting the bars)

$$\int_D(\sigma_{ij}^{(1)}u_{i,j}^{(2)} - \sigma_{ji}^{(2)}u_{i,j}^{(1)})dV = \int_{B_\sigma}(f_i^{(1)}u_i^{(2)} - f_i^{(2)}u_i^{(1)})dA + \int_{B_u}(\sigma_{ji}^{(1)}n_jg_i^{(2)} - \sigma_{ji}^{(2)}n_jg_i^{(1)})dA$$

$$+\rho\int_D(F_i^{(1)}u_i^{(2)} - F_i^{(2)}u_i^{(1)})dV \tag{25}$$

Using equation (16) and taking into consideration that $u_{i,j}^{(2)}u_{j,i}^{(1)} - u_{i,j}^{(1)}u_{j,i}^{(2)} = 0$, we obtain

$$\int_D(\sigma_{ji}^{(1)}u_{i,j}^{(2)} - \sigma_{ji}^{(2)}u_{i,j}^{(1)})dV = 2k_1\int_D(r_i^{(1)}\phi_i^{(2)} - r_i^{(2)}\phi_i^{(1)})dV$$

$$+(3\lambda_1 + 2\mu_1 + k_1)a\int_D(e^{(1)}\theta^{(2)} - e^{(2)}\theta^{(1)})dV \tag{26}$$

From equations (25) and (26) we get

$$\int_{B_\sigma}(f_i^{(1)}u_i^{(2)} - f_i^{(2)}u_i^{(1)})dA + \int_{B_u}(\sigma_{ji}^{(1)}n_jg_i^{(2)} - \sigma_{ji}^{(2)}n_jg_i^{(1)})dA + \rho\int_D(F_i^{(1)}u_i^{(2)} - F_i^{(2)}u_i^{(1)})dV$$

$$-2k_1\int_D(r_i^{(1)}\phi_i^{(2)} - r_i^{(2)}\phi_i^{(1)})dV - (3\lambda_1 + 2\mu_1 + k_1)a\int_D(e^{(1)}\theta^{(2)} - e^{(2)}\theta^{(1)})dV = 0 \tag{27}$$

Using the divergence theorem, equations $(15)_2$, (16) and (22)-(23), we get

$$\int_D (m_{ji}^{(1)}\phi_{i,j}^{(2)} - m_{ji}^{(2)}\phi_{i,j}^{(1)})dV = \int_{B_m} (\Omega_i^{(1)}\phi_i^{(2)} - \Omega_i^{(2)}\phi_i^{(1)})dA + \int_{B_\phi} (m_{ji}^{(1)}n_j\Xi_i^{(2)} - m_{ji}^{(2)}n_j\Xi_i^{(1)})dA$$

$$+\rho\int_D (M_i^{(1)}\phi_i^{(2)} - M_i^{(2)}\phi_i^{(1)})dV + 2k_1\int_D (r_i^{(1)}\phi_i^{(2)} - r_i^{(2)}\phi_i^{(1)})dV \tag{28}$$

Using equation (17) and taking into consideration that $\phi_{i,j}^{(2)}\phi_{j,i}^{(1)} - \phi_{i,j}^{(1)}\phi_{j,i}^{(2)} = 0$, we find that the integral in the left-hand side of equation (28) is equal to zero, therefore equation (27) with (28) leads to the first part of the reciprocity theorem in the Laplace transform domain

$$\int_{B_\sigma} f_i^{(1)}u_i^{(2)}dA + \int_{B_u} \sigma_{ji}^{(1)}n_j g_i^{(2)}dA + \rho\int_D F_i^{(1)}u_i^{(2)}dV + (3\lambda_1 + 2\mu_1 + k_1)a\int_D e^{(2)}\theta^{(1)}dV$$

$$+\rho\int_D M_i^{(1)}\phi_i^{(2)}dV + \int_{B_m} \Omega_i^{(1)}\phi_i^{(2)}dA + \int_{B_\phi} m_{ji}^{(1)}n_j\Xi_i^{(2)}dA = S_{21}^{12} \tag{29}$$

which contains the mechanical causes of motion F_i, f_i, M_i and Ω_i, the prescribed displacements and the surface microrotations g_i et Ξ_i. S_{21}^{12} indicates the same expression as on the left-hand side except that the superscripts (1) and (2) are interchanged.

To derive the second part we multiply $\theta^{(2)}$ by the corresponding equation (21) for the first problem, $\theta^{(1)}$ by the analogous equation for the second problem, subtracting and integrating over D, we get

$$K\int_D (\theta^{(2)}\theta_{,ii}^{(1)} - \theta^{(1)}\theta_{,ii}^{(2)})dV - (3\lambda_1 + 2\mu_1 + k_1)a(T_0 s + Nf(s,\omega))\int_D (\theta^{(2)}e^{(1)} - \theta^{(1)}e^{(2)})dV$$

$$+\int_D (Q^{(1)}\theta^{(2)} - Q^{(2)}\theta^{(1)})dV = 0 \tag{30}$$

Since $\theta^{(2)}\theta_{,ii}^{(1)} = (\theta^{(2)}\theta_{,i}^{(1)})_{,i} - \theta_{,i}^{(1)}\theta_{,i}^{(2)}$, and $\theta^{(1)}\theta_{,ii}^{(2)} = (\theta^{(1)}\theta_{,i}^{(2)})_{,i} - \theta_{,i}^{(2)}\theta_{,i}^{(1)}$, using the divergence theorem, equations (22)-(23), the left-hand side of equation (30) takes the form

$$\int_D (\theta^{(2)}\theta_{,ii}^{(1)} - \theta^{(1)}\theta_{,ii}^{(2)})dV = \int_{B_1} (\theta_{,n}^{(1)}\Phi^{(2)} - \theta_{,n}^{(2)}\Phi^{(1)})dA + \int_{B_2} (\theta^{(2)}G^{(1)} - \theta^{(1)}G^{(2)})dA \tag{31}$$

Equations (30) and (31) lead to

$$K\int_{B_1} \theta_{,n}^{(1)}\Phi^{(2)}dA + K\int_{B_2} \theta^{(2)}G^{(1)}dA + (3\lambda_1 + 2\mu_1 + k_1)a(T_0 s + Nf(s,\omega))\int_D \theta^{(1)}e^{(2)}dV$$

$$+\int_D Q^{(1)}\theta^{(2)}dV = S_{21}^{12} \tag{32}$$

Equation (32) constitutes the second part of the reciprocity theorem which contains the thermal causes of motion Φ, Q and G. Combining equations (28) and (32) we obtain the general reciprocity theorem in the Laplace transform domain

$$(T_0 s + Nf(s,\omega))\Big(\int_{B_\sigma} f_i^{(1)}u_i^{(2)}dA + \int_{B_u} \sigma_{ji}^{(1)}n_j g_i^{(2)}dA + \rho\int_D F_i^{(1)}u_i^{(2)}dV + \rho\int_D M_i^{(1)}\phi_i^{(2)}dV$$

$$+\int_{B_m} \Omega_i^{(1)}\phi_i^{(2)}dA + \int_{B_\phi} m_{ji}^{(1)}n_j\Xi_i^{(2)}dA\Big) - K\int_{B_1} \theta_{,n}^{(1)}\Phi^{(2)}dA$$

$$-K\int_{B_2} \theta^{(2)}G^{(1)}dA - \int_D Q^{(1)}\theta^{(2)}dV = S_{21}^{12} \tag{33}$$

5 Generalizations of Maysel's Formula

The problem to be solved will consist of the determination of $u_i(\mathbf{x},t)$, $\phi_i(\mathbf{x},t)$ and $\theta(\mathbf{x},t)$, $\mathbf{x} \in D,\ t > 0$, i.e. the solution of the system of equations (6)-(8), subjected to the homogeneous initial conditions (9) and (10), and the boundary conditions

$$u_i(\mathbf{x},t) = g_i(\mathbf{x},t), \quad \phi_i(\mathbf{x},t) = \Xi_i(\mathbf{x},t), \quad \theta_{,n}(\mathbf{x},t) = G(\mathbf{x},t), \quad \mathbf{x} \in B_2 = B_u = B_\phi \tag{34}$$

$$\sigma_{ij}(\mathbf{x},t)n_j(\mathbf{x}) = f_i(\mathbf{x},t), \quad m_{ij}(\mathbf{x},t)n_j(\mathbf{x}) = \Omega_i(\mathbf{x},t), \quad \theta(\mathbf{x},t) = \Phi(\mathbf{x},t), \quad \mathbf{x} \in B_1 = B_\sigma = B_m \tag{35}$$

where $g_i(\mathbf{x},t),\ \Xi_i(\mathbf{x},t),\ \Phi(\mathbf{x},t),\ f_i(\mathbf{x},t),\ \Omega_i(\mathbf{x},t)$ and $G(\mathbf{x},t)$ are functions. Consider now the three cases.

Case 1. We assume that $F_i = 0$, $M_i = 0$, and that an instantaneous source of heat located at $x_i = y_i$ where $\mathbf{y} \in (D \cup B)$, is acting upon a linear micropolar viscoelastic body, i.e., we assume $Q = Q_0\delta(r)\delta(t)$, $F_i = 0$, $M_i = 0$, where $Q_0 > 0$ is constant, $r = \sqrt{(x_i - y_i)(x_i - y_i)}$ and $\delta(...)$ is a Dirac delta function. Thus in the Laplace transform domain (omitting the bars) we have

$$Q = Q_0\delta(r), \quad F_i = 0, \quad M_i = 0 \tag{36}$$

The corresponding fundamental solutions of the system of equations (18)-(20) are

$$u_i^{(1)}, \quad \phi_i^{(1)}, \quad \theta^{(1)} \tag{37}$$

Case 2. We assume now that $Q = 0$, $M_i = 0$, and an instantaneous concentrated body force $F_i = F_i^{(j)} = F_0\delta(\mathbf{x}-\mathbf{y})\delta(t)\delta_{ij}$ is acting at the point $x_i = y_i$, where $\mathbf{y} \in (D \cup B)$, in the direction of the x_j-axis, where $F_0 > 0$ is constant. Taking the Laplace transform domain of F_i (omitting the bars) we have

$$Q = 0, \quad F_i = F_i^{(j)} = F_0\delta(r)\delta_{ij}, \quad M_i = 0 \tag{38}$$

The corresponding fundamental solutions are

$$u_i^{(j)}, \quad \phi_i^{(j)}, \quad \theta^{(j)} \tag{39}$$

Case 3. We assume now that $Q = 0$, $F_i = 0$, and an instantaneous concentrated body couple force $M_i = M_i^{(q)} = M_0\delta(\mathbf{x}-\mathbf{y})\delta(t)\delta_{iq}$ is acting at the point $x_i = y_i$ where $\mathbf{y} \in (D \cup B)$, in the direction of the x_q-axis, where $M_0 > 0$ is constant. The Laplace transform domain of M_i is

$$Q = 0, \quad F_i = 0, \quad M_i^{(q)} = M_0\delta(r)\delta_{iq} \tag{40}$$

The corresponding fundamental solutions are

$$u_i^{(q)}, \quad \phi_i^{(q)}, \quad \theta^{(q)} \tag{41}$$

Assuming the boundary conditions to be satisfied by the fundamental solutions (37), (39), and (41) in the form

$$g_i^{(l)}(\mathbf{x},s) = \Xi_i^{(l)}(\mathbf{x},s) = G^{(l)}(\mathbf{x},s), \quad \mathbf{x} \in B_2 = B_u = B_\phi \tag{42}$$

$$f_i^{(l)}(\mathbf{x},s) = \Omega_i^{(l)}(\mathbf{x},s) = \Phi^{(l)}(\mathbf{x},s), \quad \mathbf{x} \in B_1 = B_\sigma = B_m \tag{43}$$

where $l = 1, j, q$. Substituting from (34)-(35) into the reciprocity relation (33), one obtains the generalizations of Maysel's formula in the Laplace transform domain of the micropolar thermoviscoelasticity theory with non-small temperature changes

$$Q_0\theta(\mathbf{x},s) = \int_D Q\theta^{(1)}dV - (T_0 s + Nf(s,\omega))\Big(\rho\int_D F_i u_i^{(1)}dV - \int_{B_2} g_i\sigma_{ji}^{(1)}n_j dA + \int_{B_1} f_i u_i^{(1)}dA\Big)$$

$$+K\Big(\int_{B_2} G\theta^{(1)}dA - \int_{B_1}\Phi\theta_{,n}^{(1)}dA\Big) \tag{44}$$

$$F_0\rho(T_0 s+Nf(s,\omega))u_j(\mathbf{x},s)=-\int_D Q\theta^{(j)}dV+(T_0 s+Nf(s,\omega))$$

$$\Big(\rho\int_D F_i u_i^{(j)}dV+\rho\int_D M_i\phi_i^{(j)}dV+\int_{B_1} f_i u_i^{(j)}dA-\int_{B_2} g_i\sigma_{ki}^{(j)}n_k dA+\int_{B_1}\Omega_i\phi_i^{(j)}dA-\int_{B_2}\Xi_i m_{ki}^{(j)}n_k dA\Big)$$

$$-K\Big(\int_{B_2} G\theta^{(j)}dA-\int_{B_1}\Phi\theta_{,n}^{(j)}dA\Big) \tag{45}$$

$$\rho M_0\phi_q(\mathbf{x},s)=\rho\int_D F_i u_i^{(q)}dV+\rho\int_D M_i\phi_i^{(q)}dV-\int_{B_2} g_i\sigma_{ki}^{(q)}n_k dA+\int_{B_1} f_i u_i^{(q)}dA$$

$$-\int_{B_2}\Xi_i m_{ki}^{(q)}n_k dA+\int_{B_1}\Omega_i\phi_i^{(q)}dA \tag{46}$$

6 Fundamental Solutions in the Laplace Transform Domain

According to the Helmholtz theorem (Nowacki, 1962), the displacement and the body forces can be expressed in the form

$$u_i=\varphi_{,i}+\epsilon_{ijk}\Psi_{k,j},\ \ \Psi_{i,i}=0,\quad F_i=X_{,i}+\epsilon_{ijk}Y_{k,j},\ \ Y_{i,i}=0 \tag{47}$$

$$\phi_i=\Omega_{,i}+\chi_i,\ \ \chi_{i,i}=0,\quad M_i=j(Z_{,i}+N_i),\ \ N_{i,i}=0 \tag{48}$$

where $\varphi, X,\ \Omega,\ Z$ are the scalar potentials and $\Psi_k,\ Y_k,\ \chi_k,\ N_k$ are the vector potentials of the vector fields $u_i,\ F_i,\ \phi_i$ and M_i, respectively. Equations (47)-(48) with (18)-(20) lead to

$$(\nabla^2-P_1^2)\varphi-b_1\theta=-\frac{X}{C_1^2} \tag{49}$$

$$(\nabla^2-P_2^2)\Psi_i+b_2\chi_i=-\frac{Y_i}{C_2^2} \tag{50}$$

$$(\nabla^2-a_3^2)\Omega=-\frac{Z}{C_3^2} \tag{51}$$

$$(\nabla^2-a_4^2)\chi_i-b_4\nabla^2\Psi_i=-\frac{N_i}{C_4^2} \tag{52}$$

$$(\nabla^2-P^2)\theta-b\nabla^2\varphi=-Q/K \tag{53}$$

where

$$C_1^2=\frac{\lambda_1+2\mu_1+k_1}{\rho},\quad C_2^2=\frac{\mu_1+k_1}{\rho},\quad C_3^2=\frac{\alpha_1+\beta_1+\gamma_1}{\rho j},\quad C_4^2=\frac{\gamma_1}{\rho j},\quad P^2=\frac{\rho c_E s}{K}$$

$$P_n=\frac{s}{C_n}\ \ (n=1,2,3,4),\quad b_1=\frac{(3\lambda_1+2\mu_1+k_1)a}{\rho C_1^2},\quad b_2=\frac{k_1}{\mu_1+k_1},\quad b_3=\frac{2k_1}{\alpha_1+\beta_1+\gamma_1}$$

$$b=\frac{(3\lambda_1+2\mu_1+k_1)a(T_0 s+Nf(s,\omega))}{K},\quad b_4=\frac{k_1}{\gamma_1},\quad a_3^2=P_3^2+b_3,\quad a_4^2=P_4^2+2b_4$$

To obtain the fundamental solutions $u_i^{(1)},\ \phi_i^{(1)},\ \theta_i^{(1)}$ in the Laplace transform domain, we substitute equations (37) into the governing equations (49)-(53) and use the following modified Helmholtz equation (Nowacki, 1962)

$$\frac{1}{\nabla^2-m_n^2}[\delta(r)]=-\frac{1}{4\pi r}e^{-m_n r} \tag{54}$$

we obtain for an infinite region (Nowacki, 1962), taking into consideration the homogeneous initial conditions (24)

$$\Omega^{(1)} = 0, \quad \Psi_i^{(1)} = 0, \quad \chi_i^{(1)} = 0, \quad \phi_i^{(1)} = 0, \quad r_i^{(1)} = 0, \quad m_{ij} = m_{ji} = 0$$

$$\varphi^{(1)} = \frac{AQ_0 b_1}{Kr}\sum_{n=1}^{2}\aleph_n, \quad u_i^{(1)} = -\frac{AQ_0 b_1 r_{,i}}{Kr}\sum_{n=1}^{2}\wp_n, \quad \theta^{(1)} = -\frac{AQ_0}{Kr}\sum_{n=1}^{2}(m_n^2 - P_1^2)\aleph_n$$

where

$$A = \frac{1}{4\pi(m_1^2 - m_2^2)}, \quad \aleph_n = (-1)^{n-1}e^{-m_n r}, \quad \wp_n = (\frac{1}{r} + m_n)\aleph_n, \quad \varpi_n = \wp_n + r m_n^2 \aleph_n$$

$m_1^2,\ m_2^2$ are the roots of the characteristic equation

$$m^4 - (P_1^2 + b_1 b + P^2)m^2 + P_1^2 P^2 = 0$$

The fundamental solutions $u_i^{(j)}$, $\phi_i^{(j)}$, $\theta_i^{(j)}$ are obtained by substituting equations (5.6) into the governing equations (49)-(53). Taking into consideration that $\varepsilon_{ijk}Y_{k,li}^{(j)} = 0$ and $\varepsilon_{iqp}X_{,iq}^{(j)} = 0$, using equation (54) with $m_n = 0$ and equation $(47)_2$, we obtain

$$X^{(j)} = -\frac{F_0}{4\pi}(\frac{\delta_{ij}}{r})_{,i}, \quad Y_k^{(j)} = \frac{F_0}{4\pi}\varepsilon_{ijk}(\frac{\delta_{qj}}{r})_{,i}$$

The governing equations (49)-(53) now lead to

$$\varphi^{(j)} = \frac{F_0 r_{,i}\delta_{ij}}{4\pi s^2 r^2} - \frac{AF_0 r_{,i}\delta_{ij}}{r}\sum_{n=1}^{2}\frac{(m_n^2 - P^2)}{C_1^2 m_n^2}\wp_n, \quad \theta^{(j)} = -\frac{bAF_0 r_{,i}\delta_{ij}}{C_1^2 r}\sum_{n=1}^{2}\wp_n$$

$$\chi_i^{(j)} = \phi_i^{(j)} = \varepsilon_{ijk}\Big(b_4 B\frac{r_{,i}}{r}\sum_{n=3}^{4}\wp_n\Big), \quad \Psi_i^{(j)} = \varepsilon_{ijk}r_{,k}\Big(\frac{F_0}{4\pi r^2 s^2} - \sum_{n=3}^{4}\frac{B(m_n^2 - a_4^2)}{m_n^2}\wp_n\Big)$$

$$u_i^{(j)} = \frac{\hbar_1\delta_{ij}}{r^2} - \frac{\hbar_2 r_{,i}r_{,j}}{r^2}, \quad \phi_{i,i}^{(j)} = 0$$

$$\hbar_1 = -AF_0\sum_{n=1}^{2}\frac{(m_n^2 - P^2)}{C_1^2 m_n^2}\wp_n + B\sum_{n=3}^{4}\frac{(m_n^2 - a_4^2)}{m_n^2}\varpi_n$$

$$\hbar_2 = F_0\Big(B\sum_{n=3}^{4}\frac{(m_n^2 - a_4^2)}{m_n^2} - A\sum_{n=1}^{2}\frac{(m_n^2 - P^2)}{C_1^2 m_n^2}\Big)(2\wp_n + \varpi_n), \quad B = \frac{F_0}{4\pi C_2^2(m_3^2 - m_4^2)}$$

where $m_3^2,\ m_4^2$ are the roots of the characteristic equation

$$m^4 - (P_2^2 - b_2 b_4 + a_4^2)m^2 + P_2^2 a_4^2 = 0.$$

To obtain the fundamental solutions $u_i^{(q)}$, $\phi_i^{(q)}$, $\theta_i^{(q)}$, we substitute equations (39) into the governing equations (49)-(53), and we obtain for an infinite region (Nowacki, 1962), taking into consideration homogeneous initial conditions $\varphi^{(q)} = e^{(q)} = \theta^{(q)} = 0$. From equations (39) and (48) we get

$$Z^{(q)} = \frac{M_0\delta_{iq}r_{,i}}{4\pi j r^2}, \quad N_i^{(q)} = \frac{M_0}{j}\Big(\delta_{iq}\delta(r) + \frac{1}{4\pi}(\frac{1}{r})_{,iq}\Big), \quad \Omega^{(q)} = \frac{C\delta_{iq}r_{,i}}{r^2}\Big(1 - (1 + a_3 r)e^{-a_3 r}\Big)$$

$$\Omega_{,i}^{(q)} = -\frac{C(3r_{,i}r_{,q} - \delta_{iq})}{r^3}\Big(1 - (1 + a_3 r)e^{-a_3 r}\Big) + \frac{Ca_3^2 r_{,i}r_{,q}}{r}e^{-a_3 r}$$

$$\Psi_i^{(q)} = \frac{\hbar_3(3r_{,i}r_{,q} - \delta_{iq})}{r^3} - 2\frac{\ell\delta_{iq}}{3r}\sum_{n=3}^{4}\aleph_n, \quad \chi_i^{(q)} = -\frac{\hbar_4(3r_{,i}r_{,q} - \delta_{iq})}{b_2 r^3} + 2\frac{\ell\delta_{iq}}{3rb_2}\sum_{n=3}^{4}(m_n^2 - P_2^2)\aleph_n$$

$$u_i^{(q)} = \varepsilon_{ijk}\Big(\ell\frac{r_{,l}}{r}\sum_{n=3}^{4}\wp_n\Big), \quad \phi_i^{(q)} = \Omega_{,i}^{(q)} + \chi_i^{(q)}$$

where

$$\hbar_3 = \ell\Big(\frac{m_3^2 - m_4^2}{m_3^2 m_4^2} + \frac{r}{3}\sum_{n=3}^{4}\frac{\wp_n}{m_n^2}\Big), \quad C = \frac{M_0}{4\pi C_3^2 j a_3^2}, \quad \ell = \frac{M_0 b_2}{4\pi C_4^2 j(m_3^2 - m_4^2)}$$

$$\hbar_4 = \ell\Big(-\frac{m_3^2 - m_4^2}{m_3^2 m_4^2}P_2^2 + \frac{r}{3}\sum_{n=3}^{4}\frac{m_n^2 - P_2^2}{m_n^2}\wp_n\Big)$$

7 Example

In this section, the mixed boundary conditions are considered to illustrate Maysel's method. Let the problem to be determine $u_i(\mathbf{x},t)$, $\phi_i(\mathbf{x},t)$ and $\theta(\mathbf{x},t)$, $\mathbf{x}\in D,\ t>0$ the solution of the field equations (6)-(8), subjected to the homogeneous initial conditions (9)-(10) and the following boundary conditions

$$u_i(\mathbf{x},t) = g_i = 0, \;\; \phi_i(\mathbf{x},t) = \Xi_i = 0, \;\; \theta_{,n}(\mathbf{x},t) = G = 0; \;\; \mathbf{x}\in B_2 = B_u = B_\phi \tag{55}$$

$$\sigma_{ji}(\mathbf{x},t)n_j(\mathbf{x}) = f_i(\mathbf{x},t), \quad m_{ji}(\mathbf{x},t)n_j(\mathbf{x}) = \Gamma_i(\mathbf{x},t), \quad \theta(\mathbf{x},t) = \Phi(\mathbf{x},t), \;\; \mathbf{x}\in B_1 = B_\sigma = B_m \tag{56}$$

Here $f_i(\mathbf{x},t),\ \Gamma_i(\mathbf{x},t)$ and $\Phi(\mathbf{x},t)$ are given functions on $B_1(=B_\sigma = B_m)$. It is important to notice that the traction vectors $f_i(\mathbf{x},t) = \sigma_{ki}(\mathbf{x},t)n_k(\mathbf{x});\ \Gamma_i(\mathbf{x},t) = m_{ki}(\mathbf{x},t)n_k(\mathbf{x})$ and the surface temperature $\Phi(\mathbf{x},t) = \theta(\mathbf{x},t)$ are unknown functions on the part $B_2(=B_u = B_\phi)$ of the surface. The fundamental solutions $u_i^{(l)},\ \phi_i^{(l)},\ \theta^{(l)}\ (l = 1, j, q)$ satisfy the conditions

$$f_i^{(1)}(\mathbf{x},t) = f_i^{(j)}(\mathbf{x},t) = f_i^{(q)}(\mathbf{x},t) = 0, \;\; \Phi^{(1)}(\mathbf{x},t) = \Phi^{(j)}(\mathbf{x},t) = 0,$$

$$\Gamma_i^{(j)}(\mathbf{x},t) = \Gamma_i^{(q)}(\mathbf{x},t) = 0, \quad \mathbf{x}\in B_1 = B_\sigma = B_m \tag{57}$$

and since $\phi_i^{(1)} = \theta^{(q)} = 0$, we have

$$\Phi^{(q)}(\mathbf{x},t) = \Xi_i^{(1)}(\mathbf{x},t) = \Gamma_i^{(1)}(\mathbf{x},t) = 0, \; \mathbf{x}\in B_1 = B_\sigma = B_m \tag{58}$$

Equations (44)-(46) with (55)-(58) lead to the relations

$$Q_0\theta(\mathbf{x},s) = \theta_0(\mathbf{x},s) - K\int_{B_2}\theta(\mathbf{y},s)G^{(1)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})$$

$$-(T_0 s + Nf(s,\omega))\int_{B_2} f_i(\mathbf{y},s)g_i^{(1)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) \tag{59}$$

$$F_0\rho(T_0 s + Nf(s,\omega))u_j(\mathbf{x},s) = u_j^0(\mathbf{x},s) + K\int_{B_2}\theta(\mathbf{y},s)G^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})$$

$$+(T_0 s + Nf(s,\omega))\Big(\int_{B_2} f_i(\mathbf{y},s)g_i^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \int_{B_2}\Gamma_i(\mathbf{y},s)\Xi_i^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})\Big) \tag{60}$$

$$M_0\rho\phi_q(\mathbf{x},s) = \phi_q^0(\mathbf{x},s) + \int_{B_2} f_i(\mathbf{y},s)g_i^{(q)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \int_{B_2}\Gamma_i(\mathbf{y},s)\Xi_i^{(q)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) \tag{61}$$

where $\theta_0(\mathbf{x},s),\ u_j^0(\mathbf{x},s),\ \phi_q^0(\mathbf{x},s)$ are known functions, given in terms of the fundamental solutions on the part $B_1 = B_\sigma = B_m$ of the surface, functions $\Phi(\mathbf{x},s),\ f_i(\mathbf{x},s),\ \Gamma_i(\mathbf{x},s)$, the mass force, the mass-couple force and the heat source

$$\theta_0(\mathbf{x},s) = \int_D Q(\mathbf{y},s)\theta^{(1)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) - K\int_{B_1}\Phi(\mathbf{y},s)\theta^{(1)}_{,n}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})$$

$$-(T_0 s + Nf(s,\omega))\Big(\rho\int_D F_i(\mathbf{y},s)u_i^{(1)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \int_{B_1} f_i(\mathbf{y},s)u_i^{(1)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})\Big) \tag{62}$$

$$u_j^0(\mathbf{x},s) = (T_0 s + Nf(s,\omega))\Big(\rho\int_D F_i(\mathbf{y},s)u_i^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \rho\int_D M_i(\mathbf{y},s)\phi_i^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})$$

$$+\int_{B_1} f_i(\mathbf{y},s)u_i^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \int_{B_1}\Gamma_i(\mathbf{y},s)\phi_i^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})\Big) - \int_D Q(\mathbf{y},s)\theta^{(j)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})$$

$$+K\int_{B_1}\Phi(\mathbf{y},s)\theta^{(j)}_{,n}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) \tag{63}$$

$$\phi_q^0(\mathbf{x},s) = \rho\int_D F_i(\mathbf{y},s)u_i^{(q)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \rho\int_D M_i(\mathbf{y},s)\phi_i^{(q)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y})$$

$$+\int_{B_1} f_i(\mathbf{y},s)u_i^{(q)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}) + \int_{B_1}\Gamma_i(\mathbf{y},s)\phi_i^{(q)}(\mathbf{y},\mathbf{x},s)dA(\mathbf{y}). \tag{64}$$

To find the solution given by equations (59)-(61) it is necessary to determine the three unknown functions $f_i(\mathbf{x},t) = \sigma_{ki}(\mathbf{x},t)n_k(\mathbf{x})$, $\Gamma_i(\mathbf{x},t) = m_{ki}(\mathbf{x},t)n_k(\mathbf{x})$ and $\theta(\mathbf{x},t) = \Phi(\mathbf{x},t)$ on the part $B_2 (= B_u = B_\phi)$ of the surface B. In equations (59)-(61) letting $\mathbf{x} \to \xi \in B_2$ and substituting (55), we get the following system of three singular Fredholm integral equations in the three unknown functions

$$0 = \frac{\partial\theta_0(\xi,s)}{\partial n'(\xi)} - K\int_{B_2}\theta(\mathbf{y},s)\frac{\partial G^{(1)}(\mathbf{y},\xi,s)}{\partial n'(\xi)}dA(\mathbf{y})$$

$$-(T_0 s + Nf(s,\omega))\int_{B_u} f_i(\mathbf{y},s)\frac{\partial g_i^{(1)}(\mathbf{y},\xi,s)}{\partial n'(\xi)}dA(\mathbf{y}) \tag{65}$$

$$0 = u_j^{(0)}(\xi,s) + (T_0 s + Nf(s,\omega))\Big(\int_{B_2} f_i(\mathbf{y},s)g_i^{(j)}(\mathbf{y},\xi,s)dA(\mathbf{y}) + \int_{B_2}\Gamma_i(\mathbf{y},s)\Xi_i^{(j)}(\mathbf{y},\xi,s)dA(\mathbf{y})\Big)$$

$$+K\int_{B_2}\theta(\mathbf{y},s)G^{(j)}(\mathbf{y},\xi,s)dA(\mathbf{y}) \tag{66}$$

$$0 = \phi_q^{(0)}(\xi,s) + \int_{B_2} f_i(\mathbf{y},s)g_i^{(q)}(\mathbf{y},\xi,s)dA(\mathbf{y}) + \int_{B_2}\Gamma_i(\mathbf{y},s)\Xi_i^{(q)}(\mathbf{y},\xi,s)dA(\mathbf{y}) \tag{67}$$

where $n'(\xi)$ is the outward-pointed normal vector on B_2. For general boundary shapes the system of equations (59)-(61) does not seem to have analytical solutions, which makes it necessary to recur to numerical techniques. The integrals have to be discretized and the problem reduces to finding the solution of a system of linear algebraic equations.

8 Conclusions

1. The coupled term in generalized thermoelasticity formulations is modified with no-small temperature changes, and the non-linear term in heat conduction equation is considered. Instead of $T = T_0$ for the small temperature change situation, we suppose that $T = T_0 + Nt$, which can approximately simulate the temperature change in certain conditions usually met in thermo-mechanical problems, such as a finite body which is subjected to a sudden heating shock, for instance heat-treatment and laser processing. The term containing $((d/ds)\bar{u}_{i,i})$ is treated approximately depending on the assumption that $u_i(\mathbf{x},t)$ is supposed to be an odd function with respect to time.

2. The direct formulation (applying the new Betti-reciprocical theorem) of Maysel's formula in the Laplace transform domain is given for linear micropolar thermoviscoelasticity with non-small temperature changes. The new fundamental solutions for the corresponding differential equations are also obtained.

3. For the mixed boundary value problem, a system of three Fredholm integral equations in three unknown functions on a part of the boundary is obtained, and the necessity of recurring to numerical methods is shown. For any smooth enough boundary shape, the integrals involved in the system of the integral equations have to be discretized. The problem is then reduced to finding the solution of a system of linear algebraic equations in the Laplace transform domain. Using numerical inversion methods, the solutions in the physical domain can be obtained.

4. The rapid development of computer science and the boundary element applications reveal the importance of searching for a reciprocity theorem, which is the theoretical basis of the boundary element method and techniques based on Maysel's formula. The present work provides a more complete theoretical basis for modern numerical techniques such as boundary element and finite volume methods. The topic of the numerical resolution is being treated by the author and will be reported in a future paper. One can refer to Ziegler and Irschik (1987, 1985) and Ziegler (2004) for the methods of solution in thermoleasticity based on Maysel's formula and its implementation in the direct boundary integral methods.

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