

Rotating Flow of Power-Law Fluids over a Stretching Surface

M. Kumari, T. Grosan, I. Pop

The steady flow of a non-Newtonian power-law fluid due to a stretching surface in a rotating fluid has been investigated in this paper. After a similarity transformation, the set of non-linear ordinary differential equations have been solved numerically using the Keller-box method for some values of the parameter λ which is the ratio of the rotation rate to the stretching rate and the power-law index n . It is found that both the skin frictions coefficients in the x and y directions decrease with the increase of the parameter λ . However, for smaller values of λ the skin friction coefficients are higher for the dilatant fluid and smaller for the pseudoplastic fluid, respectively.

1 Introduction

The study of flow and heat transfer problems due to a stretching boundary has many practical applications in technological processes, particularly in polymer processing systems involving drawing of fibers and films or thin sheets, production of paper, linoleum, roofing shingles, insulating material, etc. Sometimes the polymer sheet is stretched while it is extruded from a die. Usually the sheet is pulled through the viscous liquid with controlled cooling system to obtain the final product with desired characteristics. The moving sheet may introduce a motion in the neighboring fluid or, alternatively, the fluid may have an independent forced-convection motion which is parallel to that of the sheet. Sakiadis (1961) was the first to investigate the flow due to a sheet issuing with constant speed from a slit into a fluid at rest. Crane (1970), and McCormack and Crane (1973) studied the steady two-dimensional incompressible boundary layer flow of a viscous and incompressible fluid caused by the stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. Since then many investigators have considered various aspects of this problem and have obtained similarity solutions and a good amount of references can be found in the papers by Magyari and Keller (1999, 2000, 2005), Liao and Pop (2004), Sparrow and Abraham (2005) and Abraham and Sparrow (2005), etc. On the other hand, the laminar incompressible boundary layer flow caused by the stretching of a flat surface in a rotating fluid has been studied by Wang (1988), Rajeswari and Nath (1992), and Nazar et al. (2004).

All of the above referenced studies have dealt with flows of Newtonian fluids. In recent years, non-Newtonian liquids have been appearing in an increasing number of applications. These applications include molten plastics, polymer solutions, dyes, varnishes, industrial suspensions, multi-grade oils, paints and printing ink, etc. Any fluid that does not behave in accordance with the Newtonian constitutive relation is called non-Newtonian. Non-Newtonian fluids have been the subject of many recent books by Astarita and Marrucci (1974), Schowalter (1978) and Slattery (1999), and in the review papers by Irvine Jr. and Karni (1987), and Andersson and Irgens (1990).

The well-known Ostwald-de-Waele power-law model has been employed on the problem of a stretching surface by Andersson and Dandapat (1991), Andersson et al. (1992, 1996), Chamkha (1997), Kumari and Nath (2001), Zheng and Zhang (2002), and Liao (2003). In this paper we consider the flow of an incompressible fluid obeying the Ostwald-de-Waele power-law model due to the stretching of a surface in a rotating fluid. The problem considered belongs to the class of two-dimensional stretching flow in a rotating fluid.

2 Basic Equations

We consider the steady laminar boundary layer flow of a non-Newtonian power-law fluid caused by the two-dimensional stretching surface in a rotating fluid. The inset of Figure 1 shows the physical model and Cartesian

coordinate system x, y, z with Ω being the angular velocity of the rotating fluid. Due to the Coriolis force, the problem considered belongs to the class of three-dimensional rotating boundary layer flows of the generalized Newtonian fluid of the power-law type. It is assumed that the stretching sheet varies linearly as a distance from the leading edge ($u_w(x) = cx, c > 0$) and the fluid is rotating with the angular velocity Ω in the z direction. The basic equations are

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} - 2 \Omega v = \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial z} \quad (2)$$

$$u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} + 2 \Omega u = \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial z} \quad (3)$$

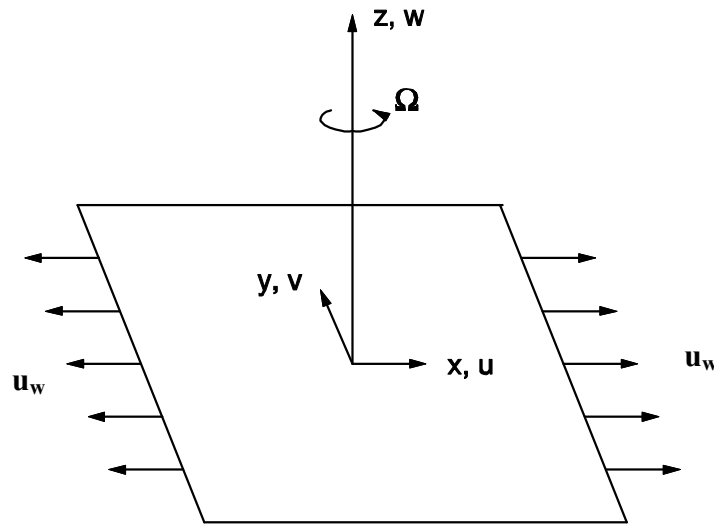


Figure 1. Physical model and coordinate system

where u, v and w are the velocity components along x -, y - and z -axes, ρ is the fluid density and the shear stress tensors τ_{xy} and τ_{xz} are defined by the Oswald-de-Waele model, see Andersson and Irgens (1990),

$$\tau_{ij} = 2K (2D_{kl} D_{kl})^{(n-1)/2} D_{ij} \quad (4)$$

Here

$$D_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (5)$$

denotes the stretching tensor, K is the consistency coefficient and n is the power-law index. The case $n = 1$ corresponds to a Newtonian fluid, $n < 1$ corresponds to a pseudo-plastic fluids and $n > 1$ corresponds to dilatant fluids, respectively. Several physical models of fluids are 23.3 % Illinois yellow clay in water ($n = 0.229$), 0.6% CMC in water ($n = 0.520$) and 10% napalm in kerosene ($n = 0.716$) for pseudoplastic fluids and ethylene oxide in sodium chloride solution ($n = 1.2, 1.6$) for dilatant fluids, see Kim et al. (1983).

Under the boundary layer approximations and the correct order-of-magnitude analysis Eqs. (2) and (3) lead to the following boundary layer equations, see Mitschka (1983),

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} - 2 \Omega v = \frac{K}{\rho} \frac{\partial}{\partial z} \left\{ \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial u}{\partial z} \right\} \quad (6)$$

$$u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} + 2 \Omega u = \frac{K}{\rho} \frac{\partial}{\partial z} \left\{ \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial v}{\partial z} \right\} \quad (7)$$

subject to the boundary conditions

$$\begin{aligned} u = u_w(x) = c x, \quad v = 0, \quad w = 0 \quad \text{at} \quad z = 0 \\ u \rightarrow 0, \quad v \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty \end{aligned} \quad (8)$$

We introduce now the following new variables

$$\begin{aligned} \eta = (\rho c^{2-n} / K)^{1/(n+1)} x^{(1-n)/(1+n)} z, \quad \psi = (\rho c^{1-2n} / K)^{-1/(n+1)} x^{2n/(n+1)} f(\eta) \\ v = c x g(\eta) \end{aligned} \quad (9)$$

where ψ is the stream function which is defined in the usual way $u = \partial\psi / \partial z$ and $w = -\partial\psi / \partial x$. Substituting (9) into Eqs. (6) and (7), we get the following set of ordinary differential equations

$$\left\{ \left[f''^2 + g'^2 \right]^{\frac{n-1}{2}} f'' \right\}' + \left(\frac{2n}{n+1} \right) f f'' - f'^2 + 2\lambda g = 0 \quad (10)$$

$$\left\{ \left[f''^2 + g'^2 \right]^{\frac{n-1}{2}} g' \right\}' + \left(\frac{2n}{n+1} \right) f g' - f' g - 2\lambda f' = 0 \quad (11)$$

and the boundary conditions (8) become

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad g(0) = 0 \\ f'(\infty) = 0, \quad g(\infty) = 0 \end{aligned} \quad (12)$$

where $\lambda = \Omega / c$ is the fluid rotation parameter and primes denote differentiation with respect to η . We notice that for $n = 1$ (Newtonian fluid), Eqs. (10) and (11) reduce to those of Wang (1988).

Physical quantities of interest are the skin friction coefficients, which are defined as

$$C_{f,xz} = \frac{\tau_{xz}}{\rho u_w^2}, \quad C_{f,yz} = \frac{\tau_{yz}}{\rho u_w^2} \quad (13)$$

where τ_{xz} and τ_{yz} are given by

$$\begin{aligned} \tau_{xz} = K \left\{ \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial u}{\partial z} \right\}_{z=0} \\ \tau_{yz} = K \left\{ \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial v}{\partial z} \right\}_{z=0} \end{aligned} \quad (14)$$

Using (9), (13) and (14), we get

$$\begin{aligned}
C_{f_{xz}} \text{Re}_x^{1/(n+1)} &= \left[(f''^2 + g'^2)^{(n-1)/2} f'' \right]_{\eta=0} \\
C_{f_{yz}} \text{Re}_x^{1/(n+1)} &= \left[(f''^2 + g'^2)^{(n-1)/2} g' \right]_{\eta=0}
\end{aligned}
\tag{15}$$

where $\text{Re}_x = (cx)^{2-n} x^n / (K/\rho)$ is the local generalized Reynolds number.

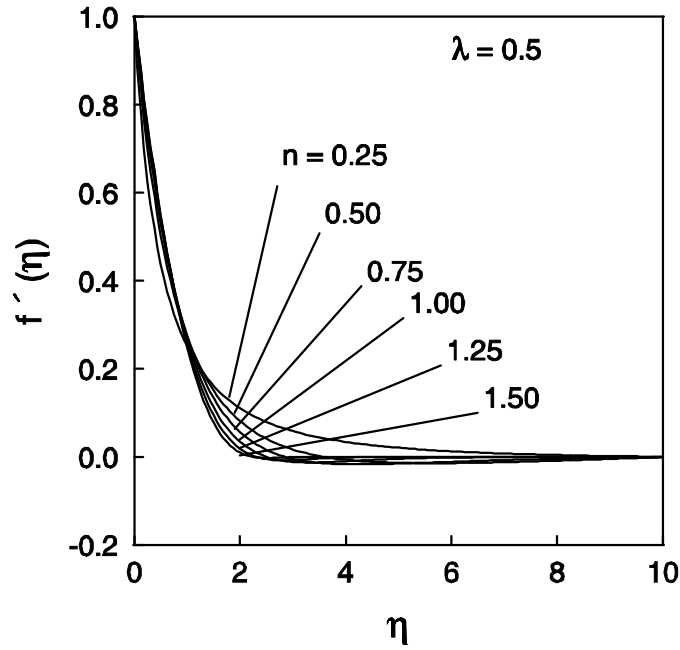


Figure 2. Velocity profiles $f'(\eta)$ in the x direction for $\lambda = 0.5$ and different values of n

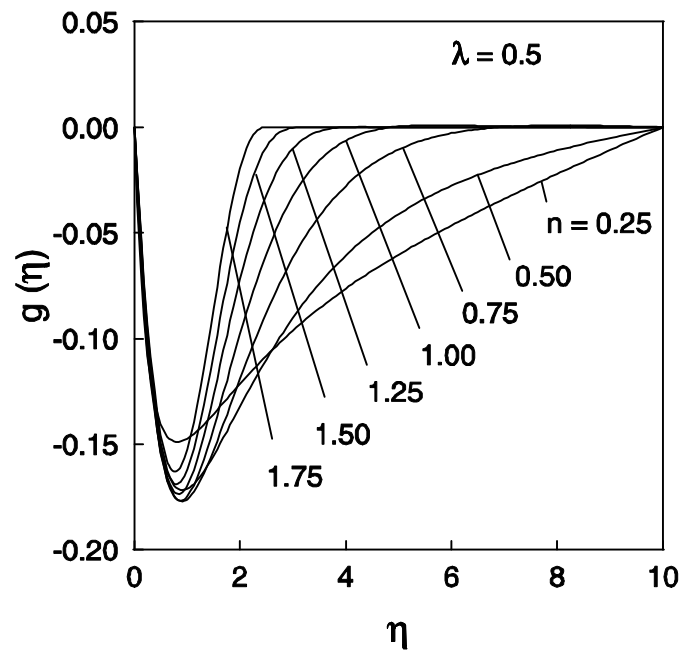


Figure 3. Velocity profiles $g(\eta)$ in the y direction for $\lambda = 0.5$ and different values of n

3 Results and Discussion

Equations (10) and (11) subject to the boundary conditions (12) have been solved numerically for $\lambda = 0.5$ and 1.0 and some values of $n = 0.25, 0.50, 1.0, 1.25, 1.50$ and 1.75 using a very efficient implicit finite-difference method known as Keller-box method which is very well described in the book by Cebeci and Bradshaw (1984). The choice of the upper limit of $n = 1.75$ was based on the fact that most power-law fluids have the value of n less than two. For the validation of the numerical method used in this study, the results for the reduced skin friction coefficients $f''(0)$ and $g'(0)$ when $n=1$ (Newtonian fluid) have been compared with those of Wang (1988), Nazar et al. (2004), and Kumari and Nath (2005). These results are given in Table 1. It is seen that the present values of $f''(0)$ and $g'(0)$ are in very good agreement with those obtained by Wang (1988), Nazar et al. (2004), and Kumari and Nath (2005). However, the results by Kumari and Nath (2005) have been obtained using an implicit finite-difference method in combination with the quasilinearization method, described in Radbill and McCue (1970) and Inouye and Tate (1974). Therefore, it can be concluded that the present results are very accurate.

λ	Wang (1988)		Nazar et al. (2004)		Kumari and Nath (2005)		Present	
	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$
0.0	-1.0000	0.0000	-1.0000	0.0000	-0.9999	0.0000	-1.00000	0.00000
0.5	-1.1384	-0.5128	-1.1384	-0.5128	-1.1382	-0.5124	-1.13838	-0.51275
1.0	-1.3250	-0.8371	-1.3250	-0.8371	-1.3251	-0.8362	-1.32503	-0.83709
2.0	-1.6523	-1.2873	-1.6523	-1.2873	-1.6535	-1.2851	-1.65235	-1.28724

Table 1. Comparison of $f''(0)$ and $g'(0)$ for a Newtonian fluid ($n = 1$) and different values of λ

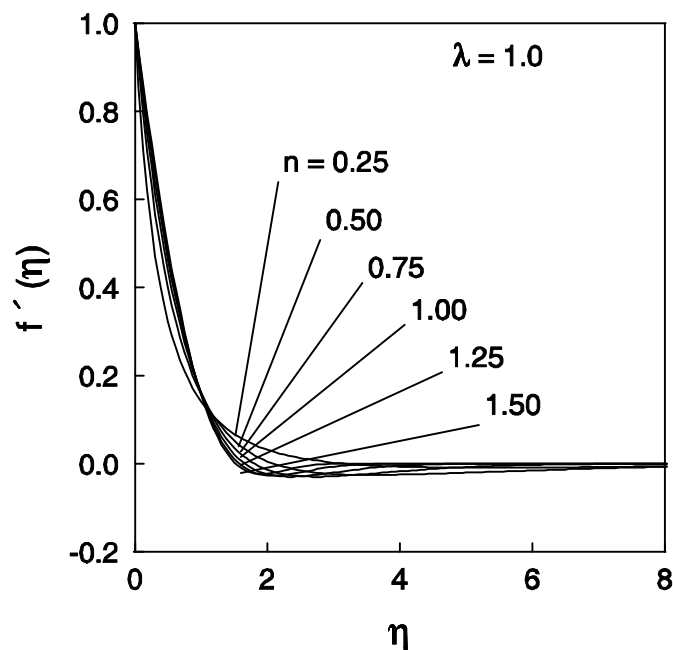


Figure 4. Velocity profiles $f'(\eta)$ in the x direction for $\lambda = 1$ and different values of n

The evolution of the similarity profiles $f'(\eta)$ and $g(\eta)$ in the x and y directions are shown for some values of n when $\lambda = 0.5$ and 1.0 in Figures 2 to 5. It is seen that with the increase of n both the boundary layer thicknesses in the x - and y - directions decrease. The dilatant fluid ($n > 1$) has a thinner boundary layer than the pseudoplastic fluid ($n < 1$) a conclusion which is in agreement with that found by Kleinstreuer and Wang (1988). However, the boundary layer thicknesses are smaller for small fluid rotation ($\lambda = 0.5$) than for higher fluid rotation ($\lambda = 1.0$), respectively. Figures 6 and 7 illustrate the variation with λ of the skin friction coefficients in the x - and y - directions for some values of n . The skin friction coefficients are measurably higher for dilatant fluids than for pseudoplastic fluids when the rotation of the fluid is slow ($\lambda \leq 0.5$) and vice versa for a higher fluid rotation ($\lambda > 0.5$), respectively.

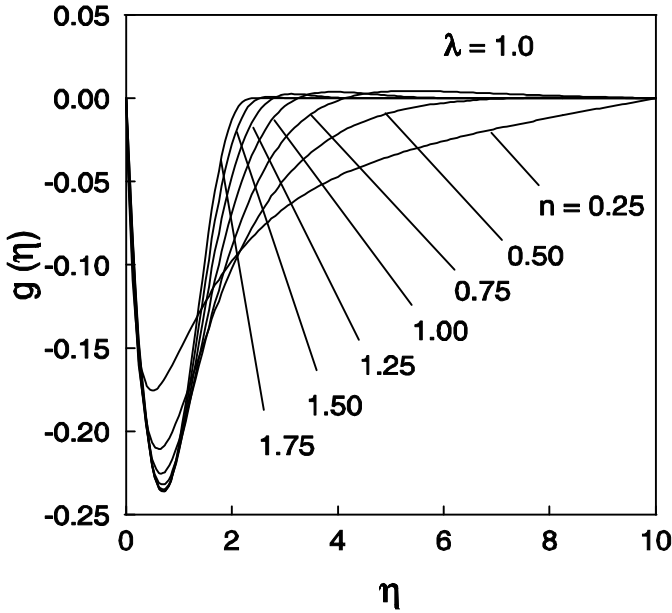


Figure 5. Velocity profiles $g(\eta)$ in the y direction for $\lambda = 1$ and different values of n

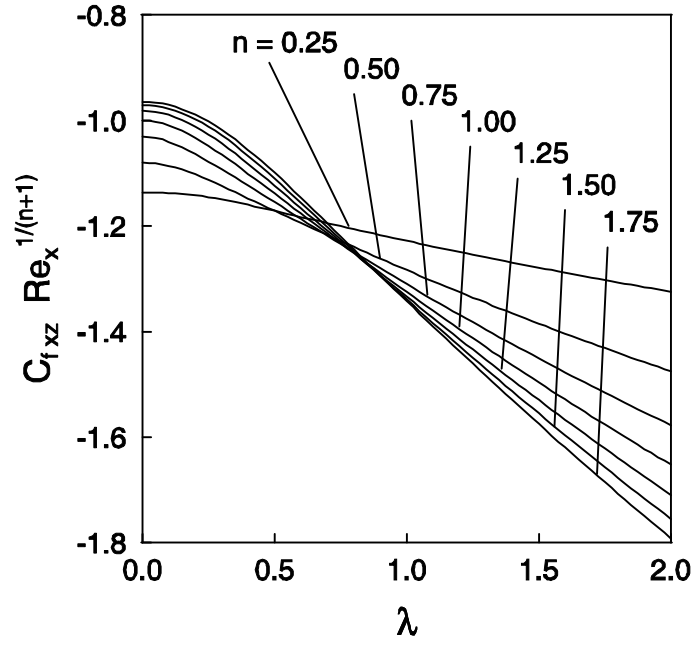


Figure 6. Variation with λ of the skin friction coefficient in the x direction for some values of n

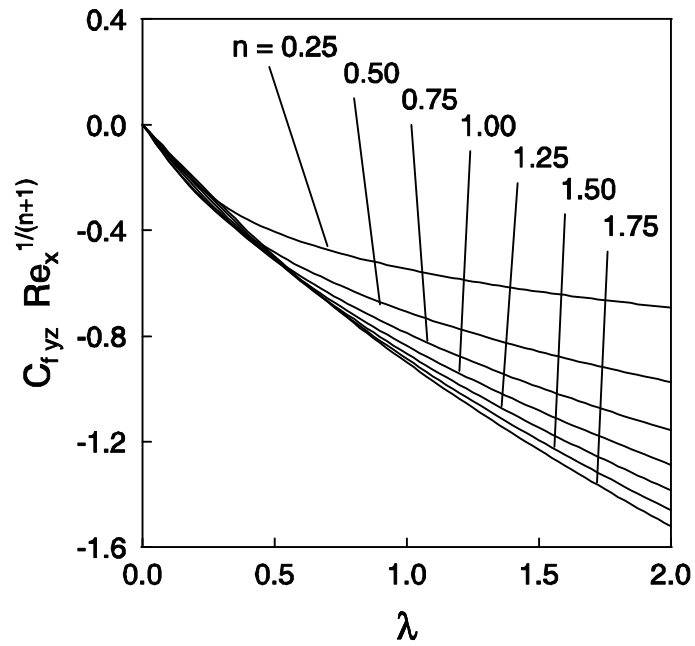


Figure 7. Variation with λ of the skin friction coefficient in the y direction for some values of n

Acknowledgement

The authors wish to express their very sincerely thanks to the referee for his valuable comments.

References

- Abraham, J.P.; Sparrow, W.M.: Friction drag resulting from the simultaneous imposed motions of a freestream and its bounding surface. *Int. J. Heat Fluid Flow*, 26, (2005), 289-295.
- Andersson, H.I.; Aarseth, J.B.; Braud, N.; Dandapat, B.S.: Flow of a power-law fluid film on an unsteady stretching surface. *J. Non-Newtonian Fluid Mech.*, 62, (1996), 1-8.
- Andersson, H.I.; Bech, K.H.; Dandapat, B.S.: Magnetohydrodynamic flow of a power-law fluid over a stretching sheet. *Int. J. Non-Linear Mech.*, 27, (1992), 929-936.
- Andersson, H.I.; Dandapat, B.S.: Flow of a power-law fluid over a stretching sheet. *Stability Appl. Anal. Continuous Media*, 1, (1991), 339-347.
- Andersson, H.I.; Irgens, F.: Film flow of power law fluids. In: *Encyclopedia of Fluid Mechanics*. Vol. 9: Polymer Flow Engineering (N.P. Cheremisinoff, ed.). Gulf Publishing Company, Houston, Texas, 617-648 (1990).
- Astarita, G.; Marrucci, G.: *Principles of Non-Newtonian Fluid Mechanics*. McGraw-Hill, London, (1974).
- Cebeci, T.; Bradshaw, P.: *Physical and Computational Aspects of Convective Heat Transfer*. Springer, New York, (1984).
- Chamkha, A.J.: Similarity solution for thermal boundary layer on a stretched surface of a non-Newtonian fluid. *Int. Comm. Heat Mass Transfer*, 24, (1997), 643-652.
- Crane, L.J.: Flow past a stretching plate. *J. Appl. Math. Phys. (ZAMP)* 21, (1970), 645-647.
- Inouye, K.; Tate, A.: Finite-difference version quasilinearization applied to boundary layer equations. *AIAA J.*, 12, (1974), 558-560.
- Irvine Jr., T.F.; Karni, J.: Non-Newtonian fluid flow and heat transfer. In: *Handbook of Single-Phase Convective Heat Transfer* (S. Kakaç, R.K. Shah and W. Aung, eds.). Wiley, New York, 20.1-20.57, (1987).
- Kim, H.W.; Jeng, D.R.; DeWitt, K.J.: Momentum and heat transfer in power-law fluid flow over two-dimensional or axisymmetric bodies. *Int. J. Heat Mass Transfer*, 26, (1983), 245-259.
- Kleinstreuer, C.; Wang, T.-Y.: Heat transfer between rotating spheres and flowing power-law fluids with suction and injection. *Int. J. Heat Fluid Flow*, 9, (1988), 328-333.
- Kumari, M.; Nath, G.: MHD boundary-layer flow of a non-Newtonian fluid over a continuous moving surface with a parallel free stream. *Acta Mechanica*, 146, (2001), 139-150.
- Kumari, M.; Nath, G.: Transient rotating flow over a moving surface with a magnetic field. *Int. J. Heat Mass Transfer*, 48, (2005), 2878-2885.
- Liao, S.J.; Pop, I.: Explicit analytic solution for similarity boundary layer equations. *Int. J. Heat Mass Transfer*, 47, (2004), 75-85.
- Liao, S.-J.: On the analytic solution of magnetohydrodynamic flows of non-Newtonian fluids over a stretching sheet. *J. Fluid Mech.*, 488, (2003), 189-212.
- Magyari, E.; Keller, B.: A direct method to calculate the heat transfer coefficient of steady similar boundary layer flows induced by continuous moving surfaces. *Int. J. Thermal Sci.*, 44, (2005), 245-254.
- Magyari, E.; Keller, B.: Exact solutions for self-similar boundary-layer flows induced by permeable stretching surfaces. *Eur. J. Mech. B-Fluids*, 19, (2000), 109-122.
- Magyari, E.; Keller, B.: Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *J. Phys. D: Appl. Phys.*, 32, (1999), 577-585.

- McCormack, P.D.; Crane, L.J.: *Physical Fluid Dynamics*. Academic Press, New York, (1973).
- Mitschka, P.: Similarity solution of combined convection heat transfer from a rotating cone or disk to non-Newtonian fluids. *ASME J. Heat Transfer*, 114, (1992), 542.
- Nazar, R.; Amin, N.; Pop, I.: Unsteady boundary layer flow due to a stretching surface in a rotating fluid. *Mech. Res. Comm.*, 31, (2004), 121-128.
- Radbill, J.R.; McCue, G.A.: *Quasilinearization in Nonlinear Problems in Fluid and Orbital Mechanics*, Elsevier, New York, (1970).
- Rajeswari, V.; Nath, G.: Unsteady flow over a stretching surface in a rotating fluid. *Int. J. Engng. Sci.*, 30, (1992), 747-756.
- Sakiadis, B.C.: Boundary-layer behavior on continuous solid surfaces. *A.I.Ch.E. J.*, 7, (1961), 26-28.
- Schowalter, W.R.: *Mechanics of Non-Newtonian Fluids*. Pergamon, Oxford, (1978).
- Slattery, J.C.: *Advanced Transport Phenomena*, Cambridge University Press, (1999).
- Sparrow, W.M.; Abraham, J.P.: Universal solutions for the streamwise variation of the temperature of a moving sheet in the presence of a moving fluid. *Int. J. Heat Mass Transfer*, 48, (2005), 3047-3056.
- Wang, C.Y.: Stretching a surface in a rotating fluid. *J. Appl. Math. Phys. (ZAMP)*, 39, (1988), 177-185.
- Zheng, L.C.; Zhang, X.X.: Skin friction and heat transfer in power-law fluid laminar boundary layer along a moving surface. *Int. J. Heat Mass Transfer*, 45, (2002), 2667-2672.

Addresses: M. Kumari, Department of Mathematics, Indian Institute of Bangalore, Bangalore, India; T. Grosan and I. Pop, Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania.
e-mail: popi@math.ubbcluj.ro