TECHNISCHE MECHANIK, Band 25, Heft 2, 100-106
Manuskripteingang: 29. November 2004

Radiation Effects on the Flow near the Stagnation Point of a Stretching Sheet

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The present paper is concerned with the study of the radiation effects (Rosseland model) on the flow of an incompressible viscous fluid over a flat sheet near the stagnation point. The system of ordinary differential equations is solved numerically using the Runge-Kutta method coupled with a shooting technique. The results show that a boundary layer is formed and its thickness increases with the radiation, velocity and temperature parameters and decreases when the Prandtl number is increased.

1 Introduction

Flow and heat transfer of an incompressible viscous fluid over a stretching sheet is present in several manufacturing processes from industry such as the extrusion of polymers, the cooling of metallic plates, the aerodynamic extrusion of plastic sheets, etc. In the glass industry, blowing, floating or spinning of fibres are processes, which involve the flow due to a stretching surface. When technological processes take place at high temperatures (cooling of a metal or glass sheet), thermal radiation effects start to play an important role and cannot be neglected (see Siegel and Howell, 1992; Modest, 2003)

Study of laminar boundary layer flow of a viscous and incompressible fluid caused by a moving rigid surface was initiated by Sakadias (1961). Later, Crane (1970) studied the flow over a linearly stretching sheet and produced a similarity solution in closed analytical form for the steady two-dimensional problem. Many authors such as Carragher and Crane (1982), Dutta et al. (1985), Elbashbeshy and Bazid (2000), Gupta and Gupta (1977) investigated the heat transfer in the flow over a stretching surface taking into account different aspects of the problem (non-newtonian fluid, uniform heat flux, temperature dependent viscosity). Other physical features such as magnetic field, viscoelasticity of the fluid, suction, three-dimensional flow have been considered by Anderssen (1992), Troy et al. (1987), Abel et al. (2002), Ariel (2003), Pop (1983), Nazar et al. (2004).

Recently, Mahapatra and Gupta (2002) studied the heat transfer in the steady two dimensional stagnation-point flow of an incompressible fluid over a stretching sheet considering the case of constant surface temperature taking into consideration the viscous dissipation of the fluid. One can find also, a very good review of this topic in the book by Pop and Ingham (2001).

In this paper, the steady two dimensional stagnation-point flow of an incompressible fluid over a stretching sheet is investigated theoretically by taking into account radiation effects using the Rosseland approximation to model the radiative heat transfer. This approximation leads to a considerable simplification in the radiation flux. The integrals are now replaced by a simple expression. The simplicity of this model, is however, offset by its approximate nature and other disadvantages (see Kumari and Nath, 2004).

2 Mathematical Model

Consider the steady, two-dimensional flow of a viscous and incompressible fluid near the stagnation point on a stretching surface placed in the plane $y = 0$ of a Cartesian system of co-ordinates Oxy $(y = 0)$ with the x-axis along the sheet. The fluid occupies the upper half plane $(y > 0)$. The stretching surface has a uniform temperature T_w and a linear velocity u_w, while the velocity of the flow external to the boundary layer is $u_e(x)$. The system of equations, which model the boundary layer flow are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u\,\frac{du}{dx}+\nu\frac{\partial\, u}{\partial y} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left[\left(\frac{16\sigma T^3}{3\kappa}+k\right)\frac{\partial T}{\partial y}\right] \tag{3}$$

where u and v are the velocity components along x - and y - directions, T is the fluid temperature, κ is the extinction coefficient, σ is the Boltzmann constant, and ρ , μ , k and c_p are the fluid density, viscosity, thermal conductivity and specific heat at constant pressure, respectively. The above equations are subject to the following boundary conditions:

$$\begin{aligned} y=0:&\quad u=u\ =bx,\ \ v=0,\ \ T=T \\ y\to\infty:&\quad u=u_e(x)=ax,\ \ T=T_\infty \end{aligned} \tag{4}$$

where *a* and *b* are constants and T_∞ is the temperature of the ambient fluid.

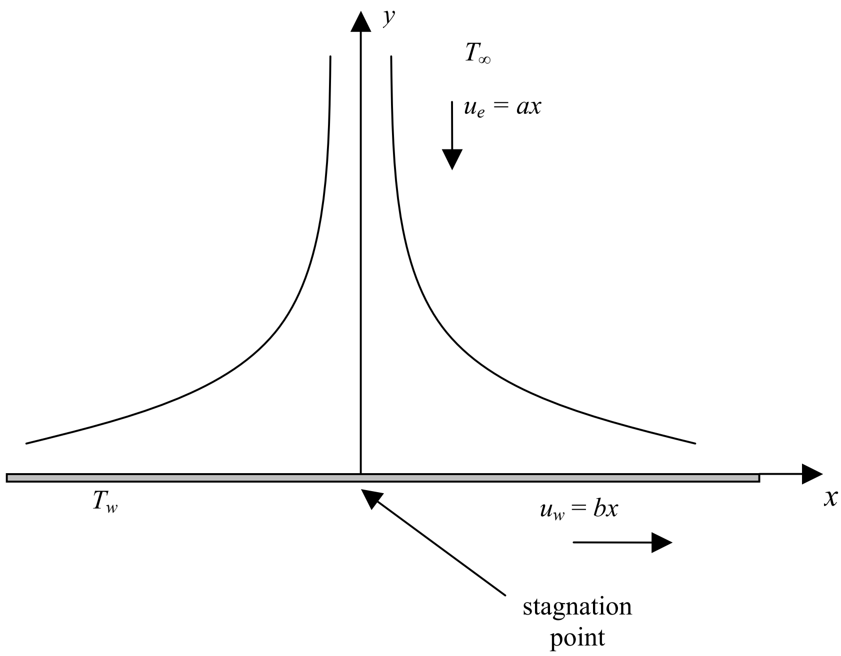

Figure 1. Physical model and co-ordinate system

Introducing the similarity variables

$$\begin{aligned} &u=bxf'(\eta),\quad v=-(b\nu)^{1/2}f(\eta) \\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \eta=\left(\frac{b}{\nu}\right)^{1/2}y\,, \end{aligned} \tag{5}$$

equations (1)-(3) can be written as

$$f'''+ff''-(f')^2+\lambda^2=0 \tag{6}$$

$$\frac{1}{Pr}\left[\left\{1+\frac{4}{3}R_d\,(1+(\theta_w-1)\theta)^3\right\}\theta'\right]+f\theta'=0 \tag{7}$$

subject to the boundary conditions (4) which become

$$f(0)=0,\ \ f'(0)=1,\ \ \theta(0)=1\,,$$

$$f'(\infty) = \lambda, \ \ \theta(\infty) = 0 \tag{8}$$

where *Pr* is the Prandtl number. Also λ , θ and R_d are the velocity, temperature and radiation parameters given by

$$\lambda = \frac{a}{b}, \ \ \theta_w = \frac{T_w}{T_\infty}, \ \ R_d = \frac{4\sigma T_\infty^3}{\kappa k}. \tag{9}$$

The physical quantities of interest in this problem are the skin friction coefficient and the local Nusselt number which can be expressed as

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2}, \quad Nu_x = \frac{x\, q_w}{k\,(T_w - T_\infty)}, \tag{10}$$

where τ_w and q_w are the skin friction and heat transfer from the sheet given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\left[\left(\frac{16\sigma T^3}{3\kappa} + k \right) \frac{\partial T}{\partial y} \right]_{y=0}, \tag{11}$$

with ρ and μ being the density and dynamic viscosity, respectively. Using variables (5) into the relations (10) and (11), we obtain

$$Re_x^{1/2}\, C_f = 2\, f\,''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = -\left(1 + \frac{4}{3} R_d \theta_w^3 \right) \theta'(0), \tag{13}$$

where $Re_x = u_w\, x / \nu$ is the local Reynolds number.

3 Results and Discussion

Equations (6) and (7), subject to the boundary conditions (8), have been solved numerically using the Runge-Kutta method coupled with a shooting technique for some values of the parameters λ , θ_w , R_d and *Pr*. We notice that in the absence of radiation effects $(R_d = 0)$, equations (6) and (7) reduce to those obtained by Mahapatra and Gupta (2002). Some obtained numerical results of $f\,''(0)$ and $\theta'(0)$ for R_d = 0 (radiation effect absent) are given in Tables 1 and 2 along with those reported by Mahapatra and Gupta (2002). It can be seen that these results are in very good agreement. However, small differences exist due to the numerical methods used. Thus, Mahapatra and Gupta (2002) solved equations (6) and (7) without radiation using the finite-differences scheme along with the Thomas algorithm, while we have used the Runge-Kutta method along with a shooting technique.

λ	$F''(0)$	
	Mahapatra and Gupta (2002)	Present results
0.1	-0.9694	-0.9694
0.2	-0.9181	-0.9181
0.5	-0.6673	-0.6673
2.0	2.0175	2.0174
3.0	4.7293	4.7290

Table 1. Values of $f''(0)$ for different values of the parameter λ in comparison with the results obtained by Mahapatra and Gupta (2002).

Further, Table 3 shows values of the Nusselt number for some values of the parameters λ, θ_w, R_d and *Pr*. We notice that for a fixed value of λ, θ_w and *Pr*, the local Nusselt number increases with the increase of the radiation parameter R_d . The reason for this trend can be explained as follows. Higher values of R_d imply lower values of the absorption coefficient κ (see the definition of R_d). Consequently, the wall heat flux given by (11) increases and hence the local Nusselt number increases. However, the Nusselt number increases with the increase of the Prandtl number. The physical reason for this trend is that a higher Prandtl number fluid has a thinner thermal boundary layer, which increases the gradient of the temperature. Consequently, the local Nusselt number is increased as *Pr* increases.

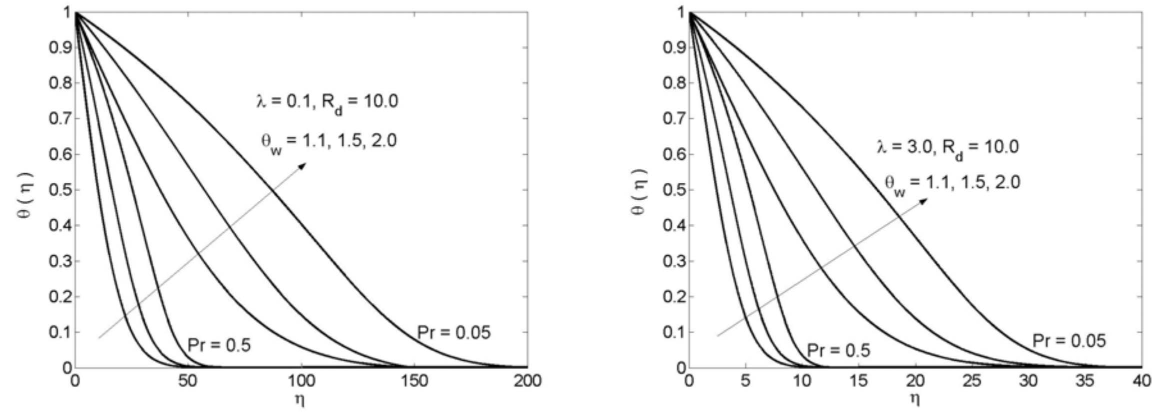

Figure 2. Variation of the temperature profile for θ_w = 1.1, 1.5, 2.0

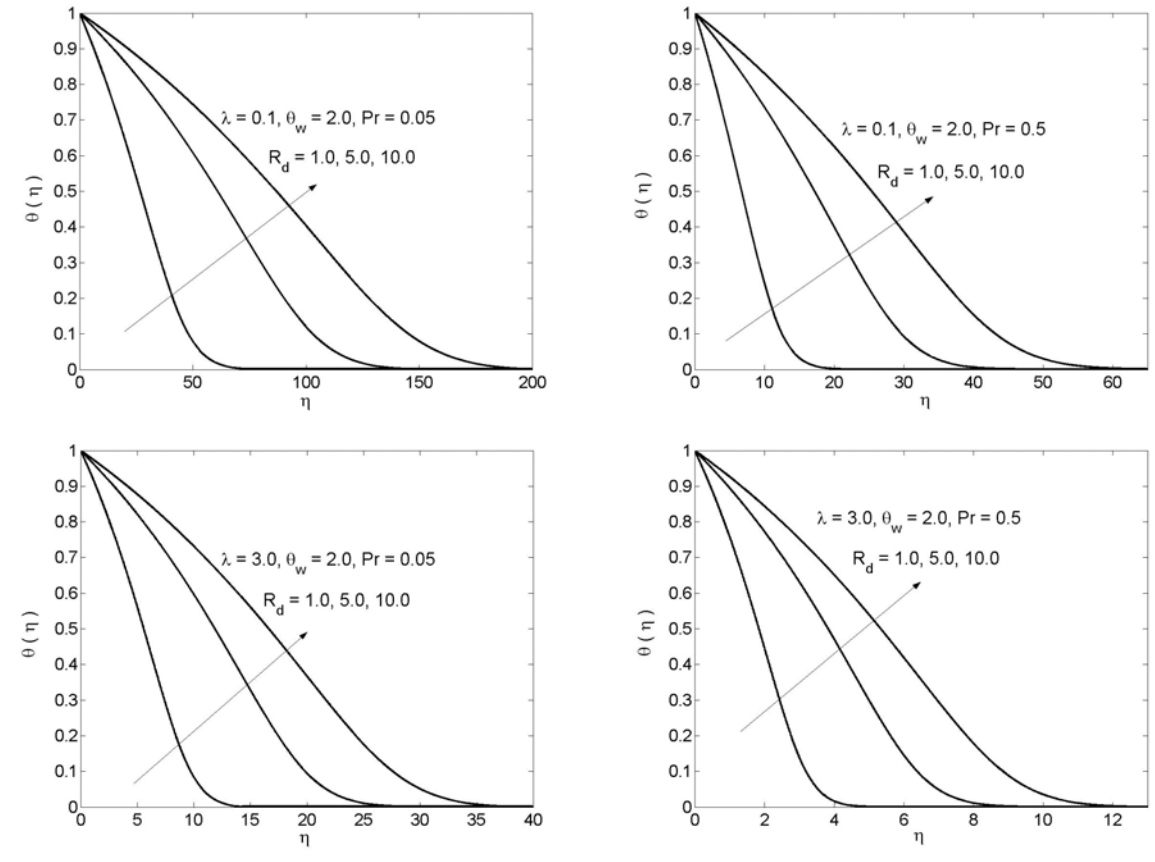

Figure 3. Variation of the temperature profiles for R_d = 1.0, 5.0, 10.0

Figures 2 to 5 show the variation of the non-dimensional temperature profiles with some values of the parameters of interest, namely, the radiation parameter R_d, Prandtl number *Pr*, velocity parameter λ , and temperature parameter θ_w. In Figure 2 it is seen that the temperature profiles became fuller and increase with the increase of the wall temperature θ_w resulting in higher surface heat flux. It is also seen that for large values of the temperature parameter the temperature profiles change their convexity. Further, the increase of the radiation parameter R_d leads to an increase of the temperature profiles and to an increase of the boundary layer thickness with R_d, as can be seen from Figure 3. Therefore, higher values of R_d imply higher surface heat flux. On the other hand, Figures 3 and 4 show, as expected, that an increase in the Prandtl number leads to a decrease of the temperature profiles. A higher Prandtl number fluid has a thinner thermal boundary layer and this increases the gradient of the temperature. Then the surface heat transfer is increased as *Pr* increases. The thermal boundary layer is embedded in the velocity (momentum) boundary layer when the Prandtl number is larger than unity. Finally, Figure 5 displays the temperature profiles for some values of the parameter λ . This figure clearly shows that the temperature profiles decrease with the increase and therefore the thinning of the thermal boundary layer with the increase of λ . This result is in complete agreement with that reported by Mahapatra and Gupta (2002).

λ	Mahapatra and Gupta (2002)				Present results			
	Pr				*Pr*			
	0.05	0.5	1.0	1.5	0.05	0.5	1.0	1.5
0.1	0.081	0.383	0.603	0.777	0.081	0.381	0.600	0.773
0.2	0.099	0.408	0.625	0.797	0.099	0.406	0.621	0.793
0.5	0.136	0.473	0.692	0.863	0.135	0.471	0.689	0.859
1.0	0.178	0.563	0.796	0.974	0.178	0.562	0.793	0.970
2.0	0.241	0.709	0.974	1.171	0.241	0.708	0.971	1.168
3.0	0.289	0.829	1.124	1.341	0.289	0.828	1.122	1.339

Table 2. Values of $-\theta'(0)$ for different values of the parameter λ in comparison with the results obtained by Mahapatra and Gupta (2002).

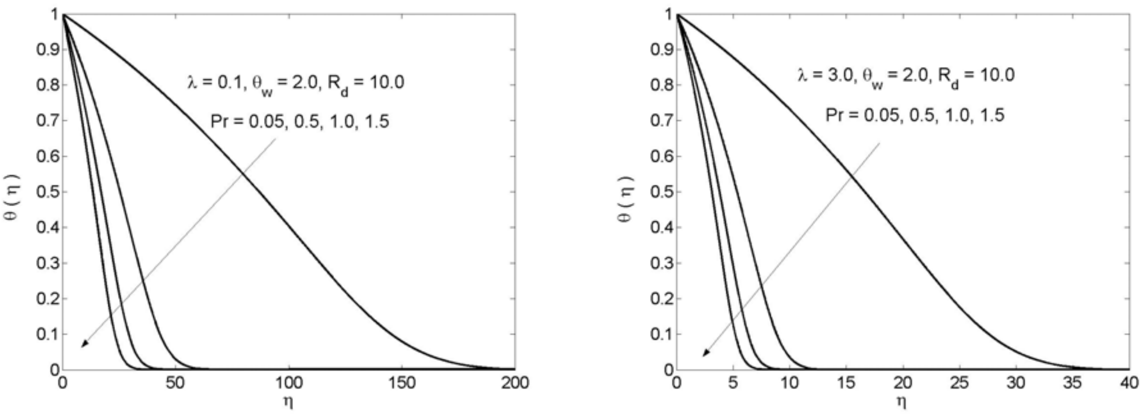

Figure 4. Variation of the temperature profile for Pr = 0.05, 0.5, 1.0, 1.5

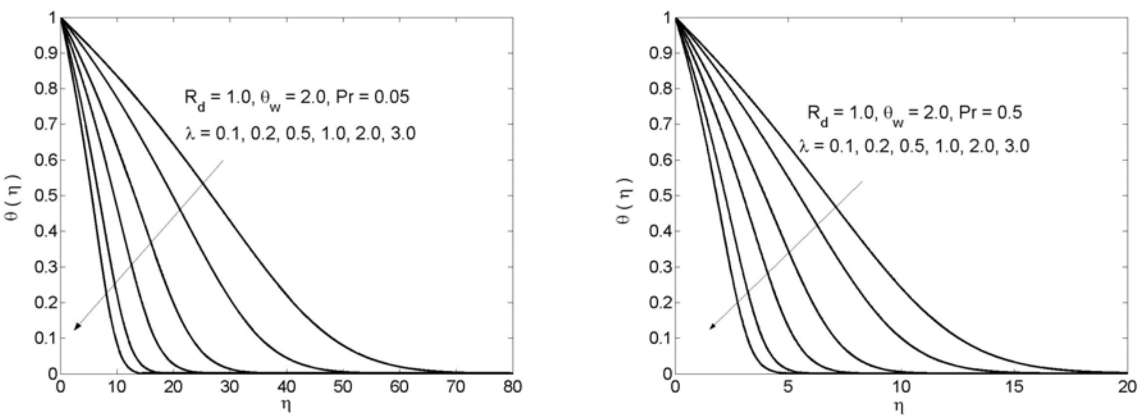

Figure 5. Variation of the temperature profile for λ = 0.1, 0.2, 0.5, 1.0, 2.0, 3.0

λ	Pr	θ_w = 1.1			θ_w = 1.5			θ_w = 2.0		
		R_d=1	R_d=5	R_d=10	R_d=1	R_d=5	R_d=10	R_d =1	R_d =5	R_d =10
0.1	0.05	0.1159	0.1965	0.2587	0.1391	0.2538	0.3496	0.1750	0.3423	0.4737
	0.5	0.5194	0.7820	0.9861	0.6022	0.9705	1.2558	0.7210	1.2442	1.6473
	1.0	0.8337	1.2391	1.5335	0.9713	1.5157	1.9182	1.1550	1.9071	2.4763
	1.5	1.0946	1.6321	2.0040	1.2848	1.9857	2.4840	1.5318	2.4667	3.1654
0.2	0.05	0.1484	0.2597	0.3524	0.1815	0.3454	0.4830	0.2322	0.4673	0.6460
	0.5	0.5830	0.0948	1.2354	0.6952	1.2173	1.6238	0.8622	1.6083	2.1856
	1.0	0.8987	1.4395	1.8522	1.0714	1.8283	2.4012	1.3183	2.3852	3.1977
	1.5	1.1568	1.8473	2.3621	1.3833	2.3335	3.0452	1.7010	3.0209	4.0160
0.5	0.05	0.2131	0.3890	0.5305	0.2656	0.5240	0.7268	0.3465	0.7226	1.0013
	0.5	0.7267	1.2904	1.7416	0.8971	1.7155	2.3552	1.1573	2.3418	3.2408
	1.0	1.0621	1.8700	2.5121	1.3079	2.4769	3.3810	1.6823	3.3632	4.6404
	1.5	1.3268	2.3281	3.1194	1.6346	3.0785	4.1906	2.0988	4.1619	5.7386
1.0	0.05	0.2888	0.5361	0.7517	0.3619	0.7285	1.0626	0.4772	1.0215	1.4104
	0.5	0.9081	1.6923	2.3283	1.1429	2.2936	3.1924	1.5085	3.1731	4.4574
	1.0	1.2833	2.3913	3.2882	1.6148	3.2430	4.5172	2.1315	4.4879	6.2985
	1.5	1.5699	2.9284	4.0268	1.9762	3.9715	5.5292	2.6098	5.4931	7.7089
2.0	0.05	0.3948	0.7454	1.0292	0.4994	1.0152	1.4214	0.6627	1.4072	1.9811
	0.5	1.1842	2.2807	3.1738	1.5114	3.1302	4.4022	2.0242	4.3738	6.1801
	1.0	1.6332	3.1703	4.4280	2.0900	4.3663	6.1594	2.8093	6.1234	8.6779
	1.5	1.9675	3.8348	5.3690	2.5196	5.2945	7.4888	3.3938	7.4437	10.5729
3.0	0.05	0.4784	0.9478	1.4229	0.6067	1.3489	1.7296	0.8085	1.7224	2.4225
	0.5	1.4051	2.7418	3.8337	1.8040	3.7788	5.3360	2.4302	5.3029	7.5259
	1.0	1.9192	3.7864	5.3203	2.4717	5.2475	7.4382	3.3472	7.3948	10.5190
	1.5	2.2958	4.5595	6.4282	2.9623	6.3379	9.0160	4.0227	8.9650	12.7800

Table 3. Values of $Nu_x / \mathrm{Re}_x^{1/2}$ for different values of the parameters λ , θ_w , R_d , Pr.

Acknowledgement

The authors wish to express their very sincere thanks to the referee for his valuable comments.

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