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Finite Element Modelling of Vibro-Acoustic Systems for Active Noise Reduction

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Over the past years a lot of scientific work has been done in the field of smart lightweight structures to reduce the structural vibrations and the radiated sound. In the paper a virtual overall model of adaptive vibro-acoustic systems, completely based on the finite element method (FEM), is presented. Beside the passive structure, this model contains the active piezoelectric elements, the acoustic fluid, the vibro-acoustic coupling, and the controller influence. According to the requirements of an effective numerical analysis of thin walled structures, the coupling between active layered shell elements and acoustic 3D hexahedron elements is implemented. It is possible to take into consideration the interior as well as the exterior radiation problem. Because of the large number of degrees of freedom of the FE model, a modal truncation technique based on a complex eigenvalue analysis is performed. After transforming the model into the state space form, the Matlab/Simulink software is used to design an appropriate controller. To show the accuracy and the performance of the developed software approach, a vibrating elastic plate and the resulting sound field are numerically investigated.

1 Introduction

The investigation of smart structural concepts stretches across a wide range of applications, such as vibration suppression, noise attenuation, shape control, damage detection and others (see e.g. Gabbert, 2002 and Tzou, 1998). In many cases structures are actively influenced by applying piezoelectric materials as distributed actuators and sensors connected with an appropriate control unit. The development and the design of such smart structures require powerful numerical analysis and simulation tools as well as suitable models including the main functional parts of the system under investigation. In vibro-acoustic systems the model should include the passive structure, the acoustic fluid, the piezoelectric sensors and actuators and the control algorithms. Such an overall virtual model can be established on the basis of the finite element method, which has to include the coupled electro-mechanical fields of the piezoelectric materials as well as the control (see Gabbert et al., 2000, 2002). Furthermore, the fluid-structure interactions have to be taken into account (see Everstine 1971). In order to solve such multi-field vibro-acoustic systems the finite element software COSAR has been extended by special 1D, 2D, 3D and layered shell-type elements with coupled mechanical and electrical degrees of freedom and brick-type finite acoustic elements as well as semi-infinite acoustic elements to study interior and exterior radiation problems, respectively. This results in large-scale finite element models of such overall vibro-acoustic smart systems, which are in general infeasible for controller design purposes. Therefore, model reduction techniques have to be applied to reduce the number of degrees of freedom (see Gabbert et al., 2002). The low frequency range modal truncation seems to be an appropriate technique. But, in vibro-acoustic systems a complex eigenvalue problem has to be solved to perform an appropriate model reduction. This requires a higher numerical effort as the solution of a positive definite eigenvalue problem. On the other hand the reduced model can also be used to calculate the transient behavior of the system in a very efficient and fast way. For controller design purposes there are special software packages available, such as Matlab/Simulink, which can be coupled with the finite element package COSAR by a special bi-directional data interface. The overall equations of motion in the time domain, generated by the FE software COSAR, are transformed into the state space form and transferred to Matlab/Simulink through a data exchange interface. In the paper the procedure is briefly explained and as a test example – the control of sound radiated from a vibrating plate is numerically studied.

2 Basic Equations and Finite Element Analysis

The theoretical background of the simulation of piezoelectric smart structures and acoustic fluids is briefly presented here. All equations are developed in a Cartesian (x_1, x_2, x_3)-coordinate system.

2.1 Finite Element Formulation of Piezoelectric Smart Structures

The derivation of a finite element formulation for the analysis of piezoelectric structures is a well known procedure, published in a lot of papers during the last years (for an overview see Gabbert et al., 2000). Based on the linear coupled electromechanical constitutive equations

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} - \mathbf{e}\mathbf{E}\,, \tag{1}$$

$$\mathbf{D} = \mathbf{e}^T\boldsymbol{\varepsilon} + \boldsymbol{\kappa}\mathbf{E}\,, \tag{2}$$

with the stress vector $\boldsymbol{\sigma}$, the vector of electric displacements $\mathbf{D}$, the elasticity matrix $\mathbf{C}$, the piezoelectric matrix $\mathbf{e}$, the dielectric matrix $\boldsymbol{\kappa}$, the strain vector $\boldsymbol{\varepsilon}$ and the electric field vector $\mathbf{E}$, the semi-discrete equations of motion of a system discretized by finite elements can be written as

$$\begin{bmatrix} \mathbf{M}_{ww} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{w}} \\ \ddot{\boldsymbol{\varphi}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{ww} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{w}} \\ \dot{\boldsymbol{\varphi}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ww} & \mathbf{K}_{w\varphi} \\ \mathbf{K}_{w\varphi}^T & -\mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\varphi} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_w \\ \mathbf{f}_\varphi \end{bmatrix}. \tag{3}$$

In equation (3) $\mathbf{w}$ contains the nodal mechanical degrees of freedom, $\boldsymbol{\varphi}$ the nodal electric potentials. $\mathbf{M}_{ww}$ is the mass matrix, $\mathbf{C}_{ww}$ the damping matrix, $\mathbf{K}_{ww}$ the stiffness matrix, $\mathbf{K}_{\varphi\varphi}$ the electric matrix, $\mathbf{K}_{w\varphi}$ the piezoelectric coupling matrix, $\mathbf{f}_w$ the mechanical load vector and $\mathbf{f}_\varphi$ the electric load vector.

2.2 Finite Element Formulation of the Acoustic Fluid

The homogeneous and inviscid acoustic fluid is modeled by using the linear acoustic wave equation (see Kollmann, 2000)

$$\frac{1}{c^2}\ddot{\Phi} - \nabla \cdot \nabla^T \Phi = 0\,, \tag{4}$$

considering the velocity potential Φ as a nodal degree of freedom. The velocity potential Φ is related to the fluid particle velocity $\mathbf{v}$ by

$$\mathbf{v} = -\nabla^T \Phi \tag{5}$$

and to the sound pressure p by

$$p = \rho_0 \dot{\Phi}\,. \tag{6}$$

The Nabla operator ∇ is defined as

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \end{bmatrix}. \tag{7}$$

Similar to the principle of virtual displacements it is possible to formulate a principle of virtual fluid potentials (Olson and Bathe, 1985). For these purposes equation (4) is multiplied by $\delta\Phi$ and integrated over the entire volume. After applying the Gaussian integral theorem and considering imposed normal velocities v_n and impedance functions $\overline{Z}$ as boundary conditions, the following result is obtained:

$$\chi = -\frac{1}{c^2}\int_{V_f} \delta\Phi\, \ddot{\Phi}\, dV - \frac{\rho_0}{\overline{Z}} \int_{O_Z} \delta\Phi\, \dot{\Phi}\, dO - \int_{V_f} \delta\Phi \nabla \cdot \nabla^T \Phi\, dV - \int_{O_v} \delta\Phi \overline{v}_n\, dO = 0 \tag{8}$$

with

$$\Phi = \overline{\Phi} \quad \text{on} \quad O_\Phi\,, \tag{9}$$

Furthermore, the whole fluid volume V_f is divided into the inner and the outer region V_i and V_o. Both volumes are connected at their interface O_k by the normal velocity boundary condition. In order to take into account the influence of the outer fluid region, which extends to infinity, we follow the Doubly Asymptotic Approximation (DAA, Geers 1978). The behavior of the outer fluid is considered only in the low and high frequency range. At low frequencies the fluid of the outer region is assumed to be incompressible. In the high frequency range, the plane waves are considered. By superimposing these two effects the following equation is obtained

$$\chi = -\frac{1}{c^2}\int_{V_f} \delta\Phi\,\ddot{\Phi}\,dV - \frac{\rho_0}{\overline{Z}}\int_{O_Z} \delta\Phi\,\dot{\Phi}\,dO - \frac{1}{c}\int_{O_k} \delta\Phi\,\dot{\Phi}\,dV - \int_{V_i} \delta\Phi\,\nabla\cdot\nabla^T\Phi\,dV$$
$$-\int_{V_o} \delta\Phi\,\nabla\cdot\nabla^T\Phi\,dV - \int_{O_v} \delta\Phi\overline{v}_n\,dO = 0\,. \tag{10}$$

Following a standard finite element procedure with approximate function for the fluid potential Φ, the matrix equation of the acoustic fluid is derived as

$$\mathbf{M}_a\ddot{\mathbf{\Phi}} + (\mathbf{C}_a + \mathbf{C}_I)\dot{\mathbf{\Phi}} + (\mathbf{K}_a + \mathbf{K}_I)\mathbf{\Phi} = \mathbf{f}_a\,, \tag{11}$$

with the acoustic mass matrix $\mathbf{M}_a$, the acoustic damping matrix $\mathbf{C}_a$, the acoustic stiffness matrix $\mathbf{K}_a$, the acoustic load vector $\mathbf{f}_a$ and the matrices $\mathbf{C}_I$ and $\mathbf{K}_I$, which characterize the outer fluid. It is important to remark that for the calculation of $\mathbf{K}_I$ a so-called semi-infinite element has been developed (see Bettess, 1992), which enables the modeling of domains having an infinite boundary at least in one direction.

2.3 Vibro-acoustic Coupling

The vibro-acoustic coupling effect results in additional loads that act on the fluid-structure interface O_S. There is load vector due to the sound pressure

$$\mathbf{f}_{wc} = \int_{O_S} \mathbf{N}_w^T\mathbf{n}p\,dO = \rho_0\int_{O_S} \mathbf{N}_w^T\mathbf{n}\mathbf{N}_a\,dO\,\dot{\mathbf{\Phi}} = \mathbf{C}_{wc}\dot{\mathbf{\Phi}} \tag{12}$$

and the load vector due to the structural vibrations

$$\mathbf{f}_{ac} = -\int_{O_S} \mathbf{N}_a^T\mathbf{n}^T\mathbf{N}_w\,dO\,\dot{\mathbf{w}} = -\frac{1}{\rho_0}\mathbf{C}_{wc}^T\dot{\mathbf{w}}\,. \tag{13}$$

The matrices $\mathbf{N}_w$ and $\mathbf{N}_a$ contain shape functions, $\mathbf{n}$ is the unity normal vector to the surface under consideration. Introducing these coupling forces, finally, the semi-discrete system of coupled equations consisting of the electro-mechanical field (see equation (3)) and the acoustic field equation (11) together with equation (12) and (13) can be written as

$$\begin{bmatrix} \mathbf{M}_{ww} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_0\mathbf{M}_a \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{w}} \\ \ddot{\boldsymbol{\varphi}} \\ \ddot{\mathbf{\Phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{ww} & \mathbf{0} & -\mathbf{C}_{wc} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{C}_{wc}^T & \mathbf{0} & -\rho_0(\mathbf{C}_a + \mathbf{C}_I) \end{bmatrix}\begin{bmatrix} \dot{\mathbf{w}} \\ \dot{\boldsymbol{\varphi}} \\ \dot{\mathbf{\Phi}} \end{bmatrix}$$
$$+\begin{bmatrix} \mathbf{K}_{ww} & \mathbf{K}_{w\varphi} & \mathbf{0} \\ \mathbf{K}_{w\varphi}^T & -\mathbf{K}_{\varphi\varphi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_0(\mathbf{K}_a + \mathbf{K}_I) \end{bmatrix}\begin{bmatrix} \mathbf{w} \\ \boldsymbol{\varphi} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_w \\ \mathbf{f}_\varphi \\ -\rho_0\mathbf{f}_a \end{bmatrix}. \tag{14}$$

2.4 Finite Element Implementation

For the analysis and the simulation of piezoelectric smart structures a number of finite elements with coupled electro-mechanical degrees of freedom are available in the FE software COSAR. For smart thin-walled structures a special layered shell type element, consisting of any number of passive and active (piezoelectric) layers (see Fig. 1a), is already available in the COSAR package (for details see Gabbert et al., 2002; Seeger, 2004). The development of this element is based on the classical *SemiLoof*-type shell element originally proposed by B. Irons (1980). Recently, new finite acoustic (see Fig. 1b) and semi-infinite acoustic finite elements (see Fig. 1c) have been developed and implemented into the COSAR software. Additionally, the coupling terms between the structural *SemiLoof* elements and the 3D acoustic finite elements have been developed and implemented, and, consequently, fully coupled vibro-acoustic simulations taking into account the interior as well as the exterior sound radiation can be performed numerically.

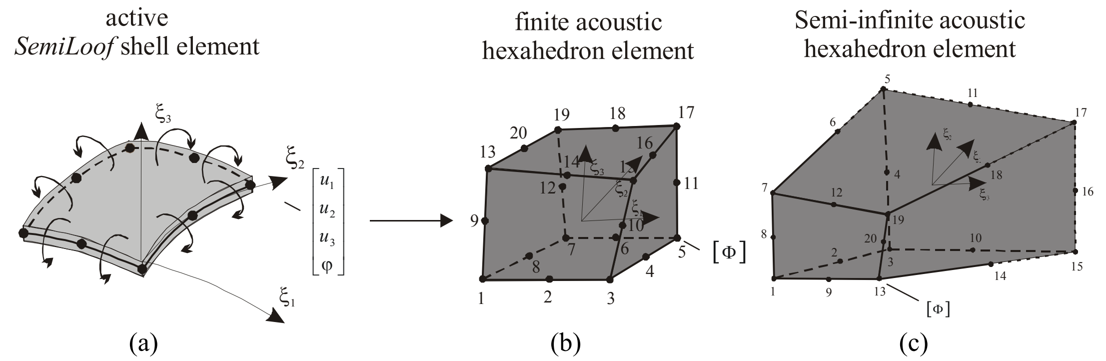

Figure 1. Coupling of shell elements and acoustic hexahedron elements: (a) piezoelectric *SemiLoof*-type shell element, (b) acoustic hexahedron element, (c) semi-infinite acoustic element

3 Model Reduction and Controller Design

As mentioned before due to the large number of degrees of freedom of a finite element model a model reduction technique is needed to design a controller. As it is well-known from the analysis of structural vibrations this reduction can be performed using a few preselected eigenmodes of the system. In the paper a modal truncation is applied, which is briefly presented in the following.

3.1 Modal Truncation

To apply the modal truncation technique to smart vibro-acoustic systems, equation (14) is rewritten in the following compact form

$$\widetilde{\mathbf{M}}\ddot{\mathbf{r}} + \widetilde{\mathbf{C}}\dot{\mathbf{r}} + \widetilde{\mathbf{K}}\mathbf{r} = \widetilde{\mathbf{f}}\ . \tag{15}$$

Introducing the state space vector

$$\mathbf{z} = \begin{bmatrix}\mathbf{r} & \dot{\mathbf{r}}\end{bmatrix}^T = \begin{bmatrix}\mathbf{w} & \boldsymbol{\varphi} & \boldsymbol{\Phi} & \dot{\mathbf{w}} & \dot{\boldsymbol{\varphi}} & \dot{\boldsymbol{\Phi}}\end{bmatrix}^T \tag{16}$$

from equation (15) it follows

$$\begin{bmatrix}\widetilde{\mathbf{C}} & \widetilde{\mathbf{M}} \\ \widetilde{\mathbf{M}} & \mathbf{0}\end{bmatrix}\dot{\mathbf{z}} + \begin{bmatrix}\widetilde{\mathbf{K}} & \mathbf{0} \\ \mathbf{0} & -\widetilde{\mathbf{M}}\end{bmatrix}\mathbf{z} = \widetilde{\mathbf{B}}\dot{\mathbf{z}} + \widetilde{\mathbf{A}}\mathbf{z} = \begin{bmatrix}\widetilde{\mathbf{f}} \\ \mathbf{0}\end{bmatrix} \tag{17}$$

From equation (16) the linear eigenvalue problem can be derived

$$\left(\widetilde{\mathbf{A}} - \lambda_i \widetilde{\mathbf{B}}\right)\hat{\mathbf{q}}_i = \mathbf{0}\ . \tag{18}$$

The solution of equation (18) results in the modal matrix $\mathbf{Q}$ with *2k* pairs of conjugate complex eigenvectors

$$\mathbf{Q} = \begin{bmatrix}\hat{\mathbf{q}}_1 & \hat{\mathbf{q}}_2 & \dots & \hat{\mathbf{q}}_{2k}\end{bmatrix}. \tag{19}$$

If the modal matrix $\mathbf{Q}$ is ortho-normalized with $\mathbf{Q}^T\widetilde{\mathbf{B}}\mathbf{Q} = \mathbf{I} = \mathrm{diag}(1)$ and $\mathbf{Q}^T\widetilde{\mathbf{A}}\mathbf{Q} = \mathbf{\Lambda} = \mathrm{diag}(\lambda_i)$, and new coordinates $\mathbf{z} = \mathbf{Qq}$ are introduced in equation (18), the reduced state space form is obtained as

$$\dot{\mathbf{q}} + \mathbf{\Lambda q} = \mathbf{Q}^T \begin{bmatrix} \widetilde{\mathbf{f}} \\ \mathbf{0} \end{bmatrix}. \tag{20}$$

3.2 Controller Design

If the state space equation (20) is extended by the measurement equation the set of equations, which can be used to design an appropriate controller is obtained.

$$\dot{\mathbf{q}} = -\mathbf{\Lambda q} + \mathbf{Q}^T \begin{bmatrix} \overline{\mathbf{B}} \\ \mathbf{0} \end{bmatrix} \mathbf{u}(t) + \mathbf{Q}^T \begin{bmatrix} \overline{\mathbf{E}} \\ \mathbf{0} \end{bmatrix} \mathbf{f}(t) = \mathbf{Aq} + \mathbf{Bu}(t) + \mathbf{Ef}(t), \tag{21a}$$

$$\mathbf{y}(t) = \mathbf{Cq} + \mathbf{Du}(t) + \mathbf{Ff}(t) \tag{21b}$$

For this reason the state matrices **A**, **B**, **E**, **C**, **D** and **F** are transferred to Matlab/Simulink via a special data exchange interface. Based on these model matrices in Matlab/Simulink a time independent LQ-controller was designed, which results in the following controller matrix **R** (for details see Nestorović et al., 2005)

$$\mathbf{u}(t) = -\mathbf{Rq}(t). \tag{22}$$

4 Example

The finite element model of a simply supported smart rectangular plate structure is used to demonstrate the capability of the presented software approach. Four piezoelectric patches as two collocated sensor/actuator pairs are attached to an elastic plate (see Fig. 2). The time-dependent behavior of the sound radiated into the upper half

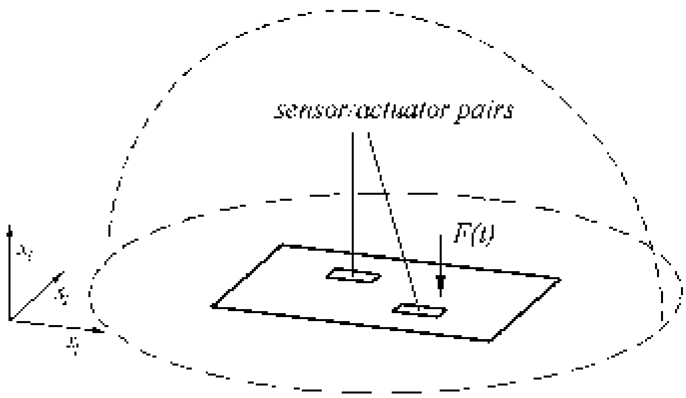

Figure 2. Smart plate structure coupled with acoustic half space

space was analyzed for the controlled and the uncontrolled case. The plate has been meshed with 96 layered *SemiLoof*-type finite shell elements. For the acoustic half space a discretization with 516 finite acoustic hexahedron elements and 236 semi-infinite acoustic hexahedron elements was performed to describe the behavior in the far field.

For controlling the system a time-independent LQ-controller was designed taking into account the first five pairs of conjugate complex eigenvectors of the coupled system. In order to demonstrate the controlled and the uncontrolled behavior the plate is excited by a harmonic force containing the first three eigenfrequencies of the system. The designed controller is switched on after 1.5 s. In Figure 3 the measured sound pressure at a distance of 639 mm above the middle of the plate is shown. After switching the controller on, the two collocated pairs of piezoelectric patch actuators and sensors result in a good noise reduction above the plate.

Table 1. Material properties and dimensions

Piezoelectric Material Properties:
$E_{11}= E_{22}=$ 60935 N/mm², $G_{12}=$ 22239 N/mm², ν=0.37,
e_{31}=-9.60·10^{-6} N/(mV)mm, κ_{33}=1.87·10^{-14} N/(mV)²,
ρ= 7.80·10^{-9} Ns²/mm^4

Material Properties of the Elastic Plate:
E=70000 N/mm², ν=0.3, ρ_P=2.63·10^{-9} Ns²/mm^4

Material Properties of the Acoustic Fluid:
c=340000 mm/s, ρ_0=1.29·10^{-12} Ns²/mm^4

Dimensions of the Plate: l_1=600 mm, l_2=400 mm, h= 2 mm

Dimensions of the Patches: 100 mm × 50 mm × 0.2 mm

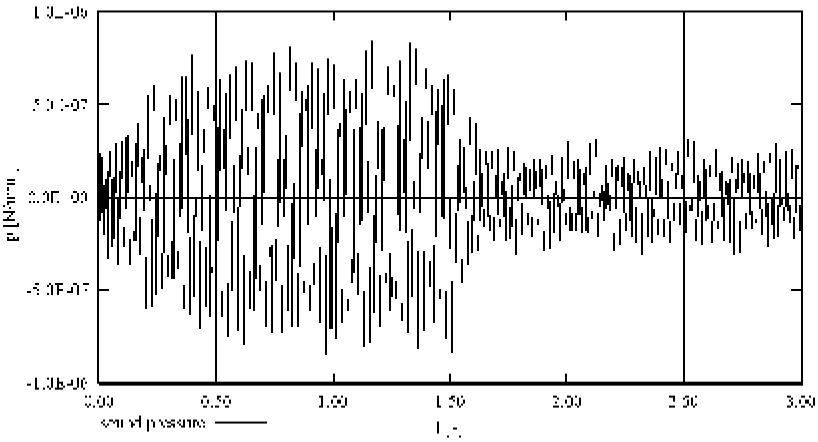

Figure 3. Resulting sound pressure

5 Conclusions

In the present paper the theoretical background of a general finite element simulation tool for the design of actively controlled thin lightweight structures to reduce the noise radiating is presented. Besides the passive structure, the finite element model includes active piezoelectric elements, the acoustic fluid, the vibro-acoustic coupling, and the controller influence. Piezoelectric layered shell type finite elements developed on the basis of the *SemiLoof* element have been extended to include a vibro-acoustic coupling with 3D acoustic finite elements and infinite elements for the far field. Because of the large number of degrees of freedom of the FE model, a modal truncation technique based on a complex eigenvalue analysis is performed in COSAR and the reduced model is transformed into the state space form. Based on a data interface the state matrices are transferred to Matlab/Simulink, where an appropriate controller can be designed and tested. The developed procedure is applied to a smart plate structure and the noise reduction after switching the controller on is presented. The efficiency of the smart vibro-acoustic system can be improved by calculating optimal positions of the actuators and sensors at the structure.

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