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Stability of a “Manipulator–Drill” System with Force Control and Time Delay

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A mathematical model of a manipulator-tool system with time delay present in the feedback loop is analyzed. The drilling process with constant feed force and control of force is the subject of this research. It is shown that the time delay in the control loop is a factor that influences the system’s stability. The minimum possible gain factor necessary for a stable system is obtained. This factor depends on the time delay and the stiffness of the sensor. The theoretically obtained results are compared to experiments.

1 Introduction

The control system of a manipulator with a tool attached to it has applications in the performance of many technological operations, for example drilling, polishing and insertion, amongst others. In all such cases, force control is necessary while the tool is working on the external object (wall, detail, bolt, etc.). Such control can be carried out using a force sensor located between the manipulator and the tool, and the properties of the control law depend on the type of technological operation (Gorinevsky et al., 1997).

A widespread technological task is drilling, with the drill firmly attached to the manipulator. Models and experiments of force control laws for the motion of manipulators during drilling are compared and analyzed in Alici and Daniel (1996), Alici (1999). The theory of drilling processes and the methods of motion control can be found in Cook (1966).
In the drilling experiments (Schmucker et al., 1996) we used a control law in which the feed rate of a drill depends on the axial loading. However, in our experiments with the “manipulator-drill” system, no attenuating oscillations due to increase of the force feedback factors were observed. Apparently, one of the possible reasons is the time delay of the control system.

The problem of stability in the motion of mechanical systems is considered in many papers, for example in Kazeroni and Tsay (1988), Kim and Zheng (1989), Kokkins (1989), Kopf (1989), Wen and Kreutz-Delgado (1992), but in most of them the influence of time delay on system stability is not examined. The stability with time delay in the manipulator control system is discussed in Gorinevsky et al. (1997), Whitney (1987), Lavrovski and Formalski (1986), Zeidis and Schneider (1999). In Alici (1999) it is experimentally shown that with time delay in the “manipulator-drill” control system, high-frequency oscillations with an increasing amplitude are observed.

In the present work, the stability of the “manipulator-drill” system is examined by the drilling process of an external workpiece. The drill is connected to the manipulator by a force sensor. The control is achieved by using a processor which receives signals from the cutting force during drilling and the position of the manipulator sensors. The mathematical model for motion of the manipulator and the drill is given, and the stability of the system is investigated with various time delays. A ratio was found, allowing an estimation of the influence of this factor on the stability of the system.

2 Mathematical Problem Statements

The one-dimensional motion of the manipulator’s bar (1) of weight m_1 incl. tool (6) of weight M_2 attached to it through the weightless sensor (5) (Figure 1) is considered. The manipulator is set into motion by the electromotor (2) through gearing (3). On the gearing’s output an output gear wheel (4) connected with a rack attached to the bar (1) is located.
The single-component sensor (5) is located between the bar and the tool (drill) including the drill bit (7) fixed in it (Figure 1). The sensor is simulated by a linear viscous elastic element.

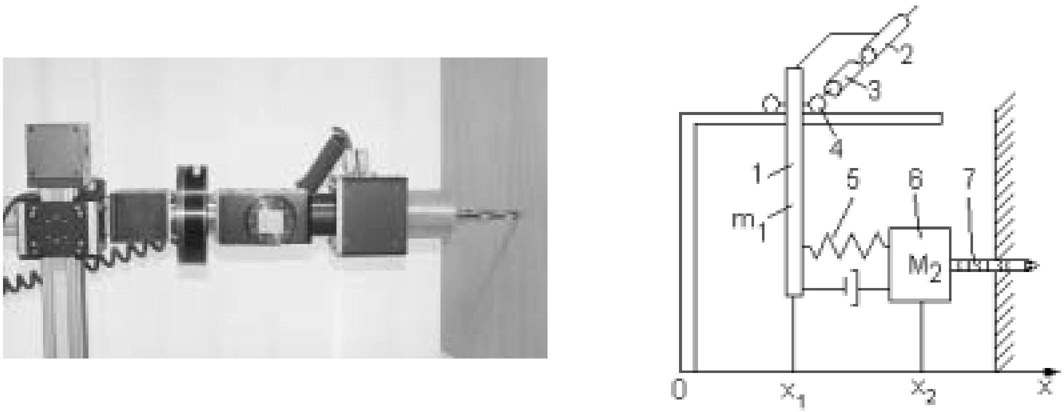


Figure 1. Experimental set-up (left) and mechanical model (right)

2.1 Equation of Motion

The coordinates of the centers of gravity of the manipulator's bar and the tool are denoted by x_1 and x_2 (Figure 1), l_0 is the distance between the centers of gravity of the bar and the tool when the elastic element of the sensor is in the unloaded state.

To obtain the equations of motion, we use the Lagrange's second-order equations

$$\frac{d}{d\,t}\left(\frac{\partial W_{kin}}{\partial \dot{q}_\nu}\right) - \frac{\partial W_{kin}}{\partial q_\nu} = Q_\nu\,, \qquad \nu = 1,\dots,n\,, \tag{1}$$

where W_{kin} is kinetic energy of the system, Q_ν are generalized forces, q_ν are generalized coordinates, and n the degree of freedom of the system.
As generalized coordinates q_ν $(\nu = 1,2)$ we take the Cartesian coordinates x_1 and x_2. The kinetic energy W_{kin} is

$$W_{kin} = \frac{1}{2}\left(m_1 \dot{x}_1^2 + M_2\,\dot{x}_2^2 + J_G \dot{\varphi}_1^2\right), \tag{2}$$

where J_G is the moment of inertia of the motor rotor. The coordinate x_1 is connected with angle φ_1 by the radius of the output gear wheel r and reduction factor j

$$x_1 = \frac{r}{j}\cdot\varphi_1 = \rho\cdot\varphi_1\,. \tag{3}$$

Thus the expression (2) for the kinetic energy takes the following form

$$W_{kin} = \frac{1}{2}\left[\left(m_1 + \frac{J_G}{\rho^2}\right)\dot{x}_1^2 + M_2\,\dot{x}_2^2\right]. \tag{4}$$

The generalized forces Q_ν shall be presented as:

$$Q_\nu = -\frac{\partial V}{\partial x_\nu} - \frac{\partial R}{\partial \dot{x}_\nu} + X_\nu, \qquad \nu = 1, 2. \tag{5}$$

The potential energy can be given in the form

$$V = \frac{1}{2} k \left(x_2 - x_1 - l_0 \right)^2, \tag{6}$$

where k is the stiffness of the sensor.
The Rayleigh function of dissipation R is determined by the expression

$$R = \frac{1}{2}\left[d_2 \dot{x}_1^2 + b(\dot{x}_2 - \dot{x}_1)^2 \right]. \tag{7}$$

The generalized force $X_1 = d_1 U$ is proportional to the voltage supplied to the motor U and does work along the virtual displacement δx_1. The generalized force $X_2 = -F_0$ is equal to the constant force of resistance during the movement of the tool into the wall and does work along the virtual displacement δx_2.
Here, d_1 and d_2 are positive constants describing the DC-motor of the manipulator's drive, b is the sensor's damping factor. The parameter d_1 shall be calculated by the nominal moment M_n as well as by the nominal voltage U_n. The parameter d_2 depends on the starting and nominal moments M_p and M_n, and also on the nominal angular speed of rotation $\dot{\varphi}_n$ (Gorinevsky et al., 1997)

$$d_1 = \frac{M_n}{U_n} \cdot \frac{1}{\rho}, \qquad d_2 = \frac{M_p - M_n}{\dot{\varphi}_n} \cdot \frac{1}{\rho^2}. \tag{8}$$

Substituting the ratio (2) - (7) into equation (1) and taking into account the expressions for the generalized forces X_1 and X_2, we obtain the equations of motion of the manipulator-tool system

$$\begin{aligned} M_1 \ddot{x}_1 + d_2 \dot{x}_1 + b\left(\dot{x}_1 - \dot{x}_2 \right) + k\left(x_1 - x_2 + l_0 \right) - d_1 U = 0, \\ M_2 \ddot{x}_2 + b\left(\dot{x}_2 - \dot{x}_1 \right) + k\left(x_2 - x_1 - l_0 \right) + F_0 = 0, \end{aligned} \tag{9}$$

with $M_1 = m_1 + \frac{J_G}{\rho^2} = \frac{J}{\rho^2}$.

2.2 Control Law

It is possible to influence the behavior of the manipulator–tool system by the voltage U submitted to the motor. The force sensor realizes the measuring of the force F. This information about F is the basis for the control law carried out by the control system. The subject of determination is the control U, which enables the stationary mode. A typical technological requirement for drilling is the constancy of the cutting speed. As follows from the formulas given in Cook's book (1966) for drilling with various diameters, the cutting speed depends linearly on the feed force. The required stationary mode is the motion of system with constant speed and a constant value of the given programming force $F_p > 0$. Thus, in this region the following conditions are satisfied

$$F = F_p = -F_0 = const, \quad \dot{x}_1 = \dot{x}_2 = \frac{F_p}{d_2} = const. \tag{10}$$

The linear law of force feedback control (Figure 2) and transfer factor $k_F > 0$ considering the time of delay T in control circuit is

$$U(t) = -k_F \left(F(t - T) - F_p \right). \tag{11}$$

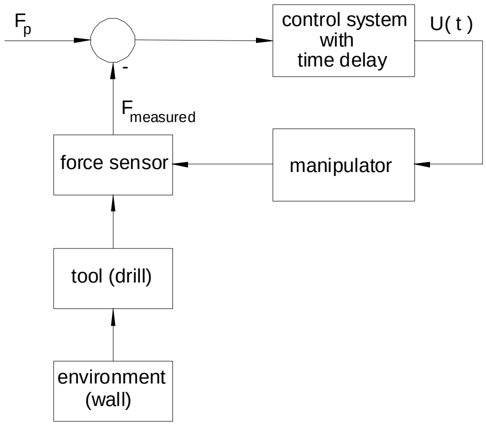


Figure 2. Control scheme

Substituting the expression for force $F(t-T)$ the equation becomes

$$U(t) = -k_F\left\{k\left[x_1(t-T) - x_2(t-T) + l_0\right] + b\left[\dot{x}_1(t-T) - \dot{x}_2(t-T)\right] - F_p\right\}. \qquad (12)$$

Equation (9) together with control law (12) describe the motion of the manipulator-tool system.

3 Stability of Stationary Mode

To analyze the stability we shall write down the equations (9) and (12) in deviations from the stationary status having kept former designations:

$$M_1\ddot{x}_1(t) + (d_2 + b)\dot{x}_1(t) + k\,x_1(t) - b\dot{x}_2(t) - k\,x_2(t) - d_1U(t) = 0\,,$$

$$M_2\ddot{x}_2(t) + b\,\dot{x}_2(t) + k\,x_2(t) - b\,\dot{x}_1(t) - k\,x_1(t) = 0\,, \qquad (13)$$

$$U(t) + k_F\,b\,\dot{x}_1(t-T) + k_F\,k\,x_1(t-T) - k_F\,b\,\dot{x}_2(t-T) - k_F\,k\,x_2(t-T) = 0\,.$$

To perform further analysis we shall pass to the dimensionless variables under the formulas

$$\bar{t} = \rho\sqrt{\frac{k}{J}}\cdot t\,, \quad \bar{x}_1 = \frac{k}{d_1U_n}x_1\,, \quad \bar{x}_2 = \frac{k}{d_1U_n}x_2\,, \quad \bar{U} = \frac{U}{U_n}\,. \qquad (14)$$

Having substituted these expressions into the system (13), we obtain the system of equations in dimensionless variables (bar will be left out):

$$\ddot{x}_1(t) + (\alpha+\beta)\dot{x}_1(t) + x_1(t) - \beta\dot{x}_2(t) - x_2(t) - U(t) = 0\,,$$

$$\mu\ddot{x}_2(t) + \beta\dot{x}_2(t) + x_2(t) - \beta\dot{x}_1(t) - x_1(t) = 0\,, \qquad (15)$$

$$U(t) + f\beta\dot{x}_1(t-\tau) + f\,x_1(t-\tau) - f\beta\dot{x}_2(t-\tau) - f\,x_2(t-\tau) = 0\,,$$

where $\alpha = \frac{d_2}{\sqrt{M_1 k}}$, $\beta = \frac{b\rho}{\sqrt{J k}}$, $\mu = \frac{M_2\rho^2}{J}$, $f = k_F\,d_1$ and $\tau = T\rho\sqrt{\frac{k}{J}}$ are positive dimensionless factors.

The characteristic equation for the system (15) is

$$K(\lambda)=\begin{vmatrix}\lambda^2+(\alpha+\beta)\lambda+1 & -(\beta\lambda+1) & -1\\ -(\beta\lambda+1) & \mu\lambda^2+\beta\lambda+1 & 0\\ f(\beta\lambda+1)\cdot e^{-\lambda\tau} & -f(\beta\lambda+1)\cdot e^{-\lambda\tau} & 1\end{vmatrix}=0\,. \tag{16}$$

For the asymptotic stability of the system (15) containing terms with time delay it is necessary and sufficient, that all roots of the quasipolynomial $K(\lambda)$ are located in the left half-plane of complex variable λ.
For small values of τ, the characteristic quasipolynomial $K(\lambda)$ can be reduced to a usual polynomial of fifth order $\mathrm{P}_5(\lambda)$, having replaced function $e^{-\lambda\tau}$ in the fraction $\dfrac{1}{\lambda\tau+1}$.
For controlled systems with pure time delay, the legitimacy of quasipolynomial replacement with a usual polynomial using the specified fraction is strictly proved in the work of Lavrovski and Formalski (1986).

The characteristic polynomial $\mathrm{P}_5(\lambda)$ can be presented as the composition $P_5(\lambda)=\lambda\cdot P_4(\lambda)$. The presence of root $\lambda=0$ in polynomial $\mathrm{P}_5(\lambda)$ (and accordingly at $K(\lambda)$) is explained by the presence in the system of equations (15) of one cyclic coordinate. Thus, the asymptotical stability should be considered in relation to four phase coordinates, which means finally to find the location of the polynomial's roots of fourth order $\mathrm{P}_4(\lambda)$ within the complex plane.
In order to locate all roots of the polynomial in the left half-plane of the complex variable λ, the necessary and sufficient conditions are given by the Hurwitz criterion (Hurwitz, 1895). This criterion is equivalent to the Nyquist criterion. The advantage of the Nyquist criterion is that it allows the determination of the stability of the closed system by the characteristic frequencies of the open loop system. Construction of the characteristic frequency of this open loop system only needs the knowledge of the characteristic frequencies of its parts. If the analytical expression of the transfer function is unknown, it can be determined experimentally. Thus, for application of the Nyquist criterion it is enough to know the experimental characteristics of the open loop system. In this case we have an analytical expression of the polynomial, therefore we shall use the Hurwitz criterion.
The polynomial $\mathrm{P}_4(\lambda)$ is:

$$\begin{aligned}\mathrm{P}_4(\lambda)=\mu\tau\cdot\lambda^4&+(\alpha\mu\tau+\beta\mu\tau+\mu+\beta\tau)\lambda^3+(\alpha\beta\tau+f\,\beta\mu+\beta+\tau+\alpha\mu+\beta\mu+\mu\tau)\lambda^2+\\&(\mu+1+\alpha\beta+\alpha\tau+f\,\mu)\lambda+\alpha\,.\end{aligned} \tag{17}$$

All factors of polynomial $\mathrm{P}_4(\lambda)$ are positive. In this case it is convenient to use the Liénard and Chipart criterion (Liénard and Chipart (1914)), allowing a reduction of twice the number of inequalities in comparison with the Hurwitz criterion.
According to this criterion of stability (find all roots with negative material parts) it is necessary and sufficient to perform the condition:

$$\Delta_3=\begin{vmatrix}\alpha\mu\tau+\beta\mu\tau+\mu+\beta\tau & \mu+1+\alpha\beta+\alpha\tau+f\,\mu & 0\\ \mu\tau & \alpha\beta\tau+f\,\beta\mu+\beta+\tau+\alpha\mu+\beta\mu+\mu\tau & \alpha\\ 0 & \alpha\mu\tau+\beta\mu\tau+\mu+\beta\tau & \mu+1+\alpha\beta+\alpha\tau+f\,\mu\end{vmatrix}>0\,. \tag{18}$$

Condition (18) is

$$A\cdot f^2+B\cdot f+C<0 \tag{19}$$

with

$$\begin{aligned}&A=\mu^3\tau-\beta^2\mu^3\tau-\beta^2\tau\mu^2-\beta\mu^3-\alpha\mu^3\tau\beta\,,\\&B=\alpha\mu^2\tau^2+\mu^2\tau+\mu^3\tau-\alpha^2\mu^3\tau-\alpha\mu^3\tau^2-4\beta^2\mu^2\tau-2\beta^2\mu^3\tau-\beta\mu^3\tau^2-2\beta\mu^2\tau^2-\beta^2\mu^2\alpha-\\&\quad 2\beta^2\tau\mu-\beta\tau^2\mu-2\beta\mu^2-2\beta\mu^3-\alpha\mu^3-\alpha^2\mu^2\tau\beta^2-3\alpha\mu^2\tau\beta-3\alpha\mu^3\tau\beta-2\alpha^2\mu^2\tau^2\beta-\\&\quad\beta^3\mu^2\tau\alpha-2\beta^2\mu^2\tau^2\alpha-\beta^3\tau\mu\alpha-2\beta^2\tau^2\alpha\mu\,,\end{aligned} \tag{20}$$

$$C=-(\beta\mu+2\beta\mu^2+\alpha\mu^3+\beta\mu^3+\beta\tau^2+\beta^2\tau+\alpha\beta\tau\mu+3\alpha\mu^2\tau\beta+2\alpha^2\mu\tau\beta^2+2\alpha\mu^3\tau\beta+$$
$$2\alpha^2\mu^2\tau^2\beta+\alpha^2\mu\tau^2\beta+\alpha^3\mu\tau^2\beta^2+\alpha^3\mu\tau^3\beta+\alpha^3\mu^2\tau\beta+2\alpha^2\mu^2\tau\beta^2+2\beta^3\mu\tau\alpha+$$
$$2\beta\mu\tau^3\alpha+2\beta^2\mu^2\tau^2\alpha+4\beta^2\mu\tau^2\alpha+\beta^3\mu\tau^2\alpha^2+\beta^2\mu\tau^3\alpha^2+\beta^3\mu^2\tau\alpha+\beta\mu^2\tau^3\alpha+$$
$$\beta^2\mu\alpha+\alpha\mu^2\tau^2+\alpha^2\mu^3\tau+\alpha\mu^3\tau^2+3\beta\mu\tau^2+3\beta^2\mu\tau+3\beta\mu^2\tau^2+3\beta^2\mu^2\tau+\beta^2\mu^3\tau+$$
$$\beta\mu^3\tau^2+\alpha^2\mu^2\beta+\beta^2\mu^2\alpha+\beta^3\tau\alpha+\beta\tau^3\alpha+2\beta^2\tau^2\alpha+\beta^3\tau^2\alpha^2+\beta^2\tau^3\alpha^2+\alpha^2\mu^2\tau^3)\,.$$

The solution of the above square inequality (19) with factors (20) for concrete values of the parameters does not present any difficulties. However the general analysis of condition (19) is complicated because of the tedious factors (20).
Considering the remarks above, we limit ourselves to two special cases.

3.1 System without Delay and with Damping in the Force Sensor

We shall examine a case in which there is no time delay, that is $\tau=0$. In this case the order of polynomial (17) can be lowered to three

$$\mathrm{P}_3(\lambda)=\mu\cdot\lambda^3+(f\,\beta\mu+\beta+\alpha\mu+\beta\mu)\lambda^2+(\mu+1+\alpha\beta+f\,\mu)\lambda+\alpha \qquad (21)$$

and the condition of asymptotic stability

$$\Delta_2=\begin{vmatrix} f\,\beta\mu+\beta+\alpha\mu+\beta\mu & \alpha \\ \mu & \mu+1+\alpha\beta+f\,\mu \end{vmatrix}= \qquad (22)$$
$$(f\,\beta\mu+\beta+\alpha\mu+\beta\mu)(\mu+\alpha\beta+f\,\mu)+f\,\beta\mu+\beta+\beta\mu>0$$

must be fulfilled for any (positive) values of the parameters. Thus, in the absence of time delay in the control loop, the manipulator-tool system is always stable.
The result obtained is similar to the conclusion made in the work of Zeidis and Schneider (1999), which showed that in the absence of time delay the system of two connected manipulators holding a common load is also always steady.

3.2 System with Delay and without Damping in the Force Sensor

The case of negligible damping in the force sensor represents the greatest practical interest.
In the absence of damping in the sensor ($\beta=0$) the factors (20) (square trinomial being in the right part of inequality (19)) become much simpler:

$$A=\mu\tau\,,$$

$$B=\alpha\tau^2+\tau+\mu\tau-\alpha^2\mu\tau-\alpha\mu\tau^2-\alpha\mu\,, \qquad (23)$$

$$C=-(\alpha\mu+\alpha\tau^2+\alpha^2\mu\tau+\alpha\mu\tau^2+\alpha^2\tau^3)\,.$$

Because factor A is positive, and C is negative, the square trinomial has two valid roots of different marks. Thus, as at $f=0$, the asymptotic stability takes place (inequality (19) is satisfied) at

$$0\le f<f^*, \qquad (24)$$

where the system behaviour is asymptotically stable. Here the critical value of the dimensionless force transfer factor f^* is equal to the positive root of the square trinomial in the left part of expression (19)

$$f^*=\frac{1}{2A}\left(-B+\sqrt{B^2-4AC}\right). \qquad (25)$$

Thus, expression (24) shows that in the presence of time delay in the control loop the stationary mode of the manipulator-tool system's motion is asymptotically stable only in the range of values limited by the above force feedback factor.

In the case of a small time delay ($\tau << 1$), it is possible to give an asymptotic expression for the critical value of the dimensionless force transfer factor f^*

$$f^* = \frac{\alpha}{\tau} + \alpha^2 - \frac{1}{\mu} \tag{26}$$

or using dimensional variables

$$k_F^* = \frac{d_2}{k\,\rho}\sqrt{\frac{J}{M_1}} \cdot \frac{1}{T} + \frac{d_2^2}{M_1\,k\,d_1} - \frac{J}{M_2\,\rho^2\,d_1}\,. \tag{27}$$

In Figure 3 examples of calculations according to expression (25) are presented. For the presentation the dependencies are indicated in dimensional variables. The curve $k_F^*(T)$ separates the region of stability and instability.

For calculations, the following values for the parameters of the manipulator, the tool and the electromechanical drive were used

Mass of the manipulator's arm $m_1 = 1\,\text{kg}$ — Mass of the tool $M_2 = 0.5\,\text{kg}$

Starting moment $M_p = 0.0147\,\text{N}\cdot\text{m}$ — Nominal torque $M_n = 0.0049\,\text{N}\cdot\text{m}$

Angular velocity of the rotor $\dot{\varphi}_n = 628\,\text{rad/s}$ — Nominal voltage $U_n = 27\,\text{V}$

Moment of inertia $J_G = 1.14 \cdot 10^{-6}\,\text{kg}\cdot\text{m}^2$ — Radius of output gear wheel $r = 5.7 \cdot 10^{-2}\,\text{m}$

Gear ratio $j = 350$

Here we find the values of the factors determining the areas of stability and instability as follows

$$\rho = 1.6 \cdot 10^{-4}\,\text{m}\,, \quad d_1 = 3.38\,\text{N/V}\,, \quad d_2 = 6.1 \cdot 10^2\ \text{N}\cdot\text{s/m}\,, \qquad M_1 = 45.5\,\text{kg}\,.$$

The dependencies presented in Figure 3 show that the area of stability decreases with growth of the sensor's rigidity (*).

The result obtained agrees with the results given in the book of Gorinevsky et al. (1997) and in the work of Zeidis and Schneider (1999).

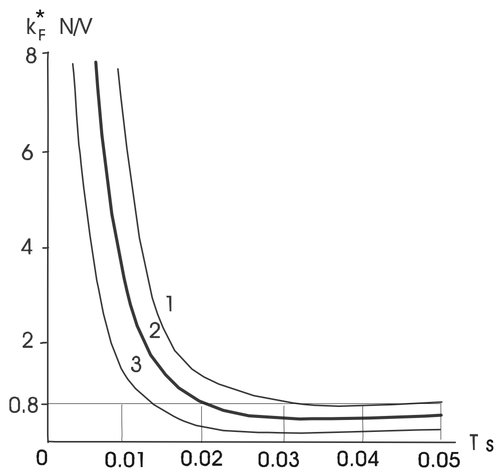


Figure 3. Limits of stability for various values of sensor's rigidity k. The area of stability is located below the curves (1– $k = 10^3\,\text{N/m}$, 2– $k = 5 \cdot 10^3\,\text{N/m}$ → parameter in the experiment, 3– $k = 10^4\,\text{N/m}$).

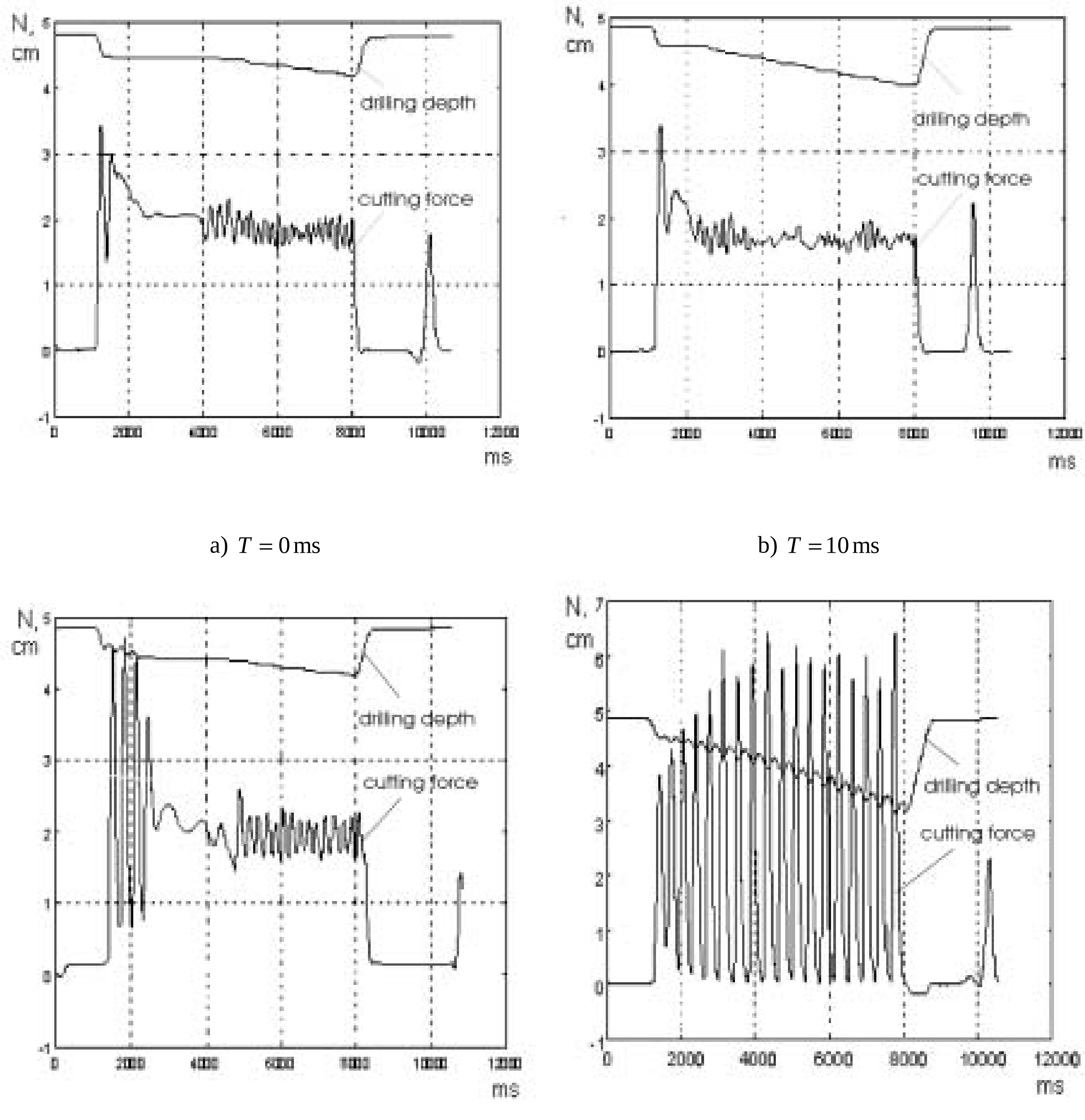
(*) V.Vatsko (student at the Otto-von-Guericke University of Magdeburg) took part in the experiments.

4 Comparison with Results of Experiment

The experiments were carried out on a manipulator using an electromechanical drive system and a drill attached to it. The system parameters are given in paragraph 3.2. Work pieces of aluminium and hardwood (oak) were chosen as the objects for drilling. The diameter of the drill was 2 mm.

The force sensor's rigidity was equal to $5 \cdot 10^3\ \mathrm{N/m}$, and the force feedback factor was $k_F = 0.8\,\mathrm{N/V}$.

The results of the experiment are indicated in Figure 4. The bottom curve shows the value of the cutting force developed by the drill during drilling, which is measured by the force sensor in N; the top curve shows the drilling depth in the workpiece in cm. The duration of the experiments was 8 seconds.



a) $T = 0\,\mathrm{ms}$

b) $T = 10\,\mathrm{ms}$

c) $T = 20\,\mathrm{ms}$

d) $T = 50\,\mathrm{ms}$

Figure 4. Results of the experiments at various time delays and for $k_F = 0.8\,\mathrm{N/V}$.

In Figure 4 (a) the specified curves are shown without time delay. In this case the vibrations in the system are quickly absorbed, which corresponds to the result received in item 3.1.

For the curves in Figures 4 (b) and 4 (c), the values of the time delay are equal to $10\,\mathrm{ms}$ and $20\,\mathrm{ms}$, respectively. In these cases the vibrations are absorbed too. The points appropriate to the force feedback factor and the specified values of time delay are located below the curve, which separates the region of stability and instability for the sensor of rigidity $5 \cdot 10^3\ \mathrm{N/m}$. So they are in the field of stability.

For the time delay of $20\,\mathrm{ms}$ the affix is close enough to the curve-separating regions of stability and instability.

For the curve in Figure 4 (d) the time delay amounts to $50\,\mathrm{ms}$. Absorption of vibrations could not be found and the point with coordinates $T = 50\,\mathrm{ms}$ and $k_F = 0.8\,\mathrm{N/V}$ lies above the curve, which separates the regions of stability and instability, in other words the point is in the region of instability.

For a complete verification it would be necessary to make a series of experiments with different parameters of coefficient k_F. Hence, the results of simulation for only one k_F confirm the theory developed in this paper.

5 Conclusions

Analysis of stability of a manipulator–tool system with time delay present in the control loop allows the following conclusions.

1. In the absence of time delay in the control loop, the manipulator–tool system is always stable.
2. With time delay present in the control loop, there are areas of stability and instability. The curves $k_F = k_F^*(T)$ separate the regions of stability and instability. With increase of sensor's rigidity the area of stability is narrowed.
3. The results of the calculations agree with the results of experiments on drilling.

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