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Contact Displacements of Spatial Contact of a Rough Body and a Body with Coating

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This paper is an example of the application of a structural nonlocal theory to the solution of the contact problem of a rough curvilinear body with plastic coating on another body. The main goal of these investigations is the creation of extrapolation methods of results of theoretical investigations from flat surfaces to any tribological system with curvilinear macro surfaces. This paper deals with the application of a phenomenological approach which is based on using the middle layer. It allows to represent a contact problem for real bodies as a superposition of two problems. The first one is the problem of penetration of rigid roughness in a plastic coating taking into account statistical characteristics of the surface. The second problem is the elastic deformation of smooth bodies with a circular area of interaction. The external load is supposed to be constant. The solution is obtained with the help of the contracting mapping principle and a simple iteration method. It is established that the first analytical approximation of solutions of the corresponding equations is useful for practical applications with sufficient accuracy. The paper deals with the case of plastic penetration of a surface asperity only when the macro contact stress is less than the yield stress of the coating, i.e. without macro plastic penetration of curvilinear bodies into this coating.

1 Introduction

Thin metal coatings are widely used in mechanical engineering. They improve the work of machine parts with lubrication and under conditions of dry friction. The appropriate selection of a metal coating allowes to provide serviceability of bearings of dry friction in deep vacuum, with high temperatures and large pressure (Alexseev, 1973; Holmberg, 1994).

The first research on friction of rigid bodies with metal coatings was carried out by F.P. Bowden. He stressed that the force of friction in the contact of bodies is defined as a product of the area by the value of the shearing stress. And if the surface of one of the bodies is covered by a thin film of soft metal, then the value of the shearing stress of such a junction is small. The force of friction decreases when the hardness of a coating is significantly less than the hardness of the bodies. This was established by experiments (Alexseev, 1973; Holmberg, 1994). At the same time the experiments demonstrated that the absolute value of the coefficient of friction for selected materials is not fixed and depends on the surface roughness of the bodies, its geometry etc.

The phenomenological approach to the problem based on reviewing the middle layer was used in the author's studies (Kravchuk, 1998; Kravchuk et al., 2000). It allowed to represent a contact problem for real bodies as a superposition of two problems. The first one is a problem of penetration of surface asperity into a plastic layer. But the mechanics of rough surfaces contains several peculiarities (Kravchuk, 1998). They are explained by the fact that the surface asperity, formed as a result of technological processing, has various heights distributions. The irregularity of the surface leads to the necessity of application of probability methods for the determination of the rigidity of the plane element of a rough surface. The similarity of dimensions of the element and the contact area explains methodical complexities of solving contact problems for real bodies (Kravchuk, 1998). The second problem is the elastic deformation of smooth bodies composed by a material in the homogenious structure.

2 General Suppositions

The height of the plastic coating is small and has no significant influence on the deformation of a body. The paper deals with the case of a plastic penetration of the surface asperity only when the macro contact stress is less than the yield stress of the coating, i.e. without macro plastic penetration of curvilinear bodies into this coating. The structural phenomenological method allows for representing interacting bodies as a system, which consists of smooth bodies separated by a middle layer. The latter consists of rigid surface asperity penetrated into the plastic coating.

An isotropic rough surface is investigated. The isotropic surface has similar probabilistic characteristics of roughness in any two directions of measurement by a profilometer. The friction is negligible in the contact area. The roughness is simulated by a set of spherical segments with a radius R_r which is shown in Figure 1. The spherical segments have a stationary value of curvature (Figure 2). The distribution of heights of segments is similar to the distribution of heights of the real roughness.

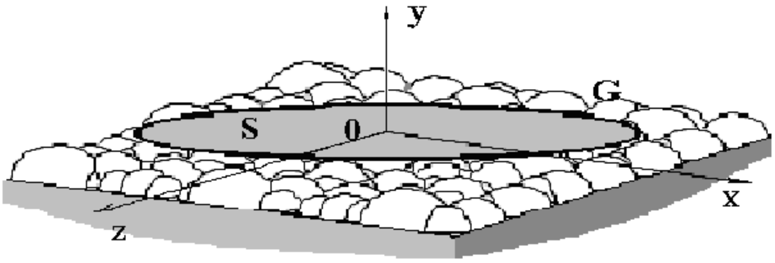

Figure 1. Model of roughness

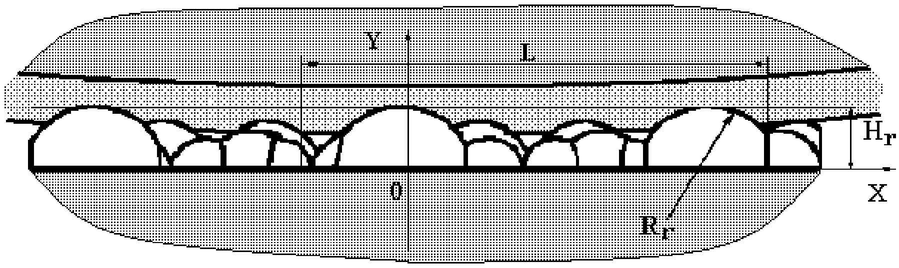

Figure 2. Plane section of bodies contact

3 Geometrical Suppositions

Let us consider the spatial contact of a flat half-space B_1 and a coated smooth ball B_2. It is supposed that the point of origin is in the point of contact of the bodies. The surface S_1 of the body B_1 has segments that are simulating the roughness. The height of the segments (roughness) is H_r (Figure 2). The height of the coating is H_c. Let us assume that H_c is a negligible value if compared with the radius of the contact area.

The average probability characteristics of the surface of a rough body are defined on some elements of the surface (profilometer trace) in any directions. The length of this trace is L (Figure 2). The base length L is always larger than the height of roughness H_r (Kravchuk et al., 2000).

The surface S_2 of a coated smooth body B_2 is defined by the equation (Johnson, 1985; Ponomarev et al., 1958)

$$y = \frac{1}{2R}\left(x^2 + z^2\right),$$

where R is the main radius of the curvature of B_2 in the point of origin.

It is supposed that $S \subset S_1$ is the statistically maximum area of contact of the bodies (Figure 1). Its bound G is a circle with the radius b in the plane $0XZ$ (Figure 1, 3). The equation of G is (Figure 3)

$$x^2 + z^2 = b^2 .$$

Let us take any point s with the coordinates (x, z) in the area of contact S (Figure 3). The distance between this point and G is not less than $L/2$. We select a cylinder C for any point (x_0, z_0, y) for which the inequality holds

$$\sqrt{(x - x_0)^2 + (z - z_0)^2} \le L/2 .$$

Hence the sub-areas $C \cap S_1$ and $C \cap S_2$ are plane and parallel circles according to the suppositions given above.

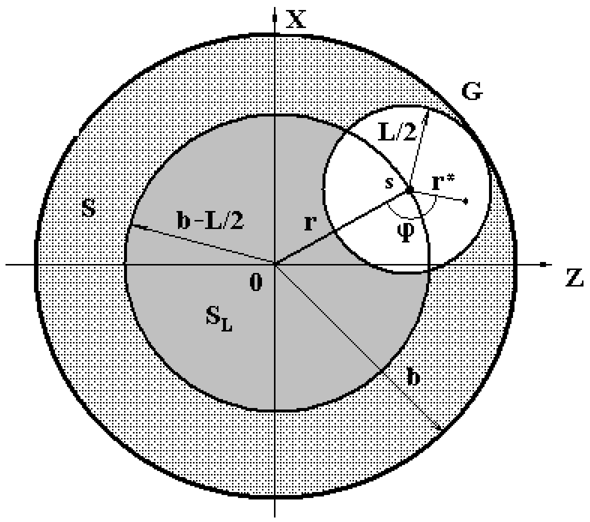

Figure 3. Projection of the contact area on the plane *XOZ*

4 Mathematical Model of Micro Level Penetration of a Surface Asperity into a Plastic Layer

The distribution of a statistically average pressure $p_L^*(x,z)$ is calculated by the equality (Kravchuk et al., 2000)

$$p_L^*(x,z) = \frac{4}{\pi L^2} \iint_{C \cap S_1} p^*(x_0, z_0)\, d\, x_0 d\, z_0$$

where $p^*(x_0, z_0)$ is a distribution of the micro pressure (pressure on peaks of segments).

The average distribution of heights between the bases of segments and the surface of the coating on $C \cap S_2$ $H_L^*(x,z)$ ($0 < \max_{(x,y) \in S_L} \{H_L^*(x,z)\} < H_r$) is calculated by the equality

$$H_L^*(x,z) = \frac{4}{\pi L^2} \iint_{C \cap S_1} H^*(x_0, z_0)\, d\, x_0 d\, z_0$$

where $H^*(x_0, z_0)$ is a distribution of micro heights (the height between the base of any segment and the surface of the coat on $C \cap S_2$).

Let us suppose that the difference of penetration of any two bases of segments in $C \cap S_1$ is small in comparison with $H_L^*(x,z)$. But the probability characteristics of roughness on any two profilometer traces with the center in any point $(x,z) \in S_L$ are equal (Figure 3). Therefore the relative approach $\varepsilon_L^*(x,z)$ of surfaces $C \cap S_1$ and $C \cap S_2$ determined by the penetration has the following form

$$\varepsilon_L^*(x,z) = \left(H_r - H_L^*(x,z)\right)/H_r$$

Let us use the following equation for the definition of a connection between $\varepsilon_L^*(x,z)$ and $p_L^*(x,z)$ (Alexseev, 1973; Kravchuk et al., 2000)

$$\varepsilon_L^*(x,z) = \left(\frac{p_L^*(x,z)}{3.4\ \sigma_s\ b_r} \right)^{\frac{1}{\lambda}} \left(1 + O\left\{ \frac{1}{\lambda} \left(0.7 \frac{H_c}{R_r} \right)^{\lambda} \right\} \right) \tag{1}$$

where σ_s is the yield stress of the coating, λ, b_r are the coefficients of bearing at a curve ($\lambda > 1$) which are determined with the help of a profilometer. The area of the real contact of a rough body and the body with coating is the area square of the real contact peaks of roughness. It is significantly smaller than the nominal contact area. Therefore in (1) it is supposed that the penetration of nearby peaks has no sufficient influence on each other (Alexseev, 1973).

5 Connection of Contact Macro-Pressure and Correction of Macro-Displacements

The average macro-pressure $p_L(x,z)$ acts on the bases of segments in the direction of the body B_1. Hence the following equality holds for the statistically average value $p_L^*(x,z)$ and the average macro-pressure $p_L(x,z)$ for any point $(x,z) \in S_L$ (Kravchuk et al., 2000)

$$p_L^*(x,z) = p_L(x,z),$$

where

$$p_L(x,z) = \frac{4}{\pi L^2} \iint_{C \cap S_1} p(x_0, z_0)\ d\,x_0 d\,z_0\ . \tag{2}$$

$p(x_0, z_0)$ is the distribution of contact macro-pressures, i.e. a solution of the contact problem for smooth bodies.

On the other hand the normal contact macro-displacement $\mathrm{V}_{B_1}^n(x,z)$ in any point $(x,z) \in S \subset S_1$ is represented as a superposition

$$\mathrm{V}_{B_1}^n(x,z) = \mathrm{V}_{S_1}^n(x,z) + \mathrm{V}_R(x,z)\ , \tag{3}$$

where $\mathrm{V}_{S_1}^n(x,z)$ is a normal contact displacement of the smooth surface of B_1, $\mathrm{V}_R(x,z)$ is a displacement which is determined by the penetration of segments (roughness) into the coating. The last function is the solution of a contact problem with a homogeneous layer and it can be represented as $\mathrm{V}_R(x,z) = H_r \cdot \varepsilon_R(x,z)$. Hence we can determine the contact average macro-deformation $\varepsilon_{R,L}(x,z)$ as (Kravchuk et al., 2000)

$$\varepsilon_{R,L}(x,z) = \frac{4}{\pi L^2} \iint_{C \cap S_1} \varepsilon_R(x_0, z_0)\ d\,x_0 d\,z_0 \tag{4}$$

The function $\mathrm{V}_{B_1}^n(x,z)$ is a solution of an initial contact problem only if the following equality is fulfilled for any point $(x,z) \in S_L$

$$\varepsilon_{R,\,L}(x,z) = \varepsilon_L^*(x,z)\ .$$

Therefore we can use the same nonlinear equation (1) for the definition of the connection between the macro-deformation $\varepsilon_{R,L}(x,z)$ and the macro-pressure $p_L(x,z)$ for any point $(x,z) \in S_L$.

6 Approximate Definition of the Contact Displacement and Pressure in the Contact Problem for Spheres (Analogue Hertz's Theory)

The contact displacement has the following form (Johnson, 1985; Ponomarev et al., 1958)

$$V_{B_1}^n + V_{S_2}^n = \delta - \left(\frac{r^2}{2R}\right) \tag{5}$$

where (Figure 3)

$$r = \sqrt{x^2 + z^2} \ ,$$

δ is a maximal contact displacement, $V_{B_1}^n$ is a contact displacement of the body B_1.
But the displacement of the half-space $V_{B_1}^n$ is defined by (3). Hence we obtain from equation (5) that

$$V_{S_1}^n + V_{S_2}^n = \delta - \left(\frac{r^2}{2R}\right) - V_R \ . \tag{6}$$

We use the following equation for an approximate evaluation of V_R

$$V_R(r) = H_r \frac{\Delta}{2}\left(\alpha\, b^2 - r^2\right) \tag{7}$$

where $\alpha \geq 1$ and Δ are coefficients which homogenize the deformation properties of a rough surface, *b* is the radius of a circle contact. We will determine the coefficient of homogenization later.

On the other hand we obtain from equations (4), (7) that

$$\varepsilon_{R,L}(r) = \frac{\Delta}{2}\left(\alpha\, b^2 - F_1(r)\right), \tag{8}$$

where (Figure 3)

$$F_1(r) = \frac{4}{\pi L^2} \int_0^{2\pi} \left(\int_0^{L/2} \left(\left(r - r^* \cos(\varphi)\right)^2 + \left(r^* \sin(\varphi)\right)^2 \right) r^*\, dr^* \right) d\varphi$$

The contact macro-pressure is of the following form taking account (6) and (7)

$$p(r) = \frac{3}{2}\frac{P}{\pi b^2}\sqrt{\left(1 - \frac{r^2}{b^2}\right)} \tag{9}$$

where P is a value of the load. Hence we obtain from (2) and (9) that

$$p_L(r) = \frac{3P}{2\,\pi\, b^2} F_2(r,b)\ , \tag{10}$$

where

$$F_2(r,b) = \frac{4}{\pi L^2} \int_0^{2\pi} \left(\int_0^{L/2} \sqrt{1 - \frac{\left(r - r^* \cos(\varphi)\right)^2 + \left(r^* \sin(\varphi)\right)^2}{b^2}}\ r^*\, dr^* \right) d\varphi \ .$$

7 Definition of the Homogenization Coefficient Δ

We determine the coefficient Δ from (7) taking account a precise fulfillment of (1) in the point of origin. We obtain from (8) and (10) that:

$$\Delta = 2\left(\frac{1}{3.4\ \sigma_s\ b_r}\frac{3P}{2\pi b^2}\right)^{\frac{1}{\lambda}}\frac{F_2(0,b)^{\frac{1}{\lambda}}}{\alpha b^2 - F_1(0)} \quad . \tag{11}$$

But the folowing equations are valid

$$\frac{F_1(0)}{\alpha b^2} = -\frac{L^2}{8\alpha b^2}$$

$$F_2(0,b) = 1 - \frac{1}{16}\frac{L^2}{b^2} + o\left(\frac{L^2}{16 b^2}\right) .$$

The problem is well posed only when $\alpha \geq 1$. To simplify the following calculations for a circular area of contact, we suppose that $b \geq 1.5L$. Hence we obtain from (11)

$$\Delta \approx 2\left(\frac{1}{3.4\ \sigma_s\ b_r}\frac{3P}{2\pi b^2}\right)^{\frac{1}{\lambda}}\frac{1}{\alpha b^2} \quad . \tag{12}$$

8 Definition of the Homogenization Coefficient α

We define the coefficient α from the condition of minimization of the average squared error of approximation of (1). We obtain the following equation by substituting (11) in (8) and taking account (1) and (12)

$$X(\alpha,b) = \frac{1}{b - L/2}\int_0^{b-L/2}\left(\frac{1}{\alpha b^2}\cdot\frac{\alpha b^2 - F_1(0)}{\left(F_2(r,b)\right)^{1/\lambda}} - 1\right)^2 dr \tag{13}$$

We obtain from (13)

$$X(\alpha,b) = \left(A(b) - \frac{B(b)}{\alpha} + \frac{C(b)}{\alpha^2}\right) \tag{14}$$

where

$$A(b) = \frac{1}{b - L/2}\int_0^{b-L/2}\left\{\frac{1}{\left(F_2(r,b)\right)^{1/\lambda}} - 1\right\}^2 dr ,$$

$$B(b) = 2\ \frac{1}{b - L/2}\int_0^{b-L/2}\left\{\frac{1}{\left(F_2(r,b)\right)^{1/\lambda}} - 1\right\}\frac{F_1(r)}{b^2\left(F_2(r,b)\right)^{1/\lambda}}dr ,$$

$$\tilde{N}(b) = \frac{1}{b - L/2}\int_0^{b-L/2}\left\{\frac{F_1(r,0)}{b^2\left(F_2(r,b)\right)^{1/\lambda}}\right\}^2 dr .$$

A s s e r t i o n. Function $X(\alpha,b)$ (14) has a single minimum for any b at

$$\alpha = \frac{2\,C(b)}{B(b)} \tag{15}$$

and it is fulfilled when $\alpha \geq 1$.

P r o o f. This assertion is based on the following facts.

i. The functions $F_{i,i=1,2} > 0$ and $0 < F_2(r,b) \leq 1$ for any $r < b - L/2$. Hence the inequalities $A(b) > 0$, $B(b) > 0$ and $C(b) > 0$ are fulfilled for any fixed b.

ii. The function $X(\alpha,b)$ has two suspicious points $\alpha = 0$ and $\alpha = \dfrac{2\,C(b)}{B(b)} > 0$. But the function $X(\alpha,b)$ at the first point $\alpha = 0$ is unlimited.

iii. An investigation of the asymptotic behavior for any point α_1 of the small neighborhood of $\alpha = \dfrac{2\,C(b)}{B(b)}$ shows that $X(\alpha_1,b) \geq X(\alpha,b)$.

iv. The inequality $\alpha \geq 1$ is the outcome of the items i., iii. and the inequality

$$\left(A(b) - \frac{B(b)}{\alpha} + \frac{C(b)}{\alpha^2} \right) \leq \left(A(b) - B(b) + C(b) \right).$$

The assertion is proved. The value of $X(\alpha,b)$ allows us to determine the accuracy of the approximation of (1).

9 Definition of the Contact Displacement for a Circle Area of Contact

The circle area of contact S is defined by the radius b (Johnson, 1985; Ponomarev et al., 1958). The combination of equations (5), (6), (12) and (15) leads to the following form of the maximal displacement

$$\delta = \frac{3\,P}{4\,E^*\,b} + H_r \left(\frac{1}{3.4\ \sigma_s\ b_r} \frac{3P}{2\,\pi\,b^2} \right)^{\frac{1}{\lambda}} \tag{16}$$

$$E^* = \left(\frac{1-\nu_1{}^2}{E_1} + \frac{1-\nu_2{}^2}{E_2} \right)^{-1}$$

where b is given by the nonlinear equation

$$b = f(b) \tag{17}$$

$$f(b) = \sqrt[3]{\frac{3}{4}\frac{R\,P}{E^*} + H_r R \left(\frac{1}{3.4\,\sigma_s\,b_r} \frac{3P}{2\,\pi\,b^2} \right)^{\frac{1}{\lambda}} \frac{B(b)}{C(b)}\, b} \quad .$$

The equations (16), (17) are valid for smooth bodies if $H_r = 0$. We have used the method of simple iteration for solving (17). Its root is supposed to belong to a segment

$$\mathbf{B} = \left[b_0, b_0 + \frac{q}{(1-q)} \left| b_0 - f(b_0,) \right| \right]$$

where

$$b_0 = \sqrt[3]{\frac{3}{4}\frac{R\,P}{E^*}}$$

$$q = \max_B \left\{ \left| \frac{d}{db} f(b) \right| \right\} < 1 \,. \qquad (18)$$

The results of a numerical analysis of equations (17) and (18) show that $q < 0{,}3$. The error ratio of the approximate definition of the contact area with the help of $b_1 = f(b_0)$ is less than 0.05.

The maximal contact displacement for the interaction of coated and rough bodies is less than the same parameters of smooth bodies without coating (Figure 4). The characteristics of roughness and plastic coating have an essential influence on the size of the contact area for small loads. The average squared error ratio of satisfying condition (1) is less than 0.04.

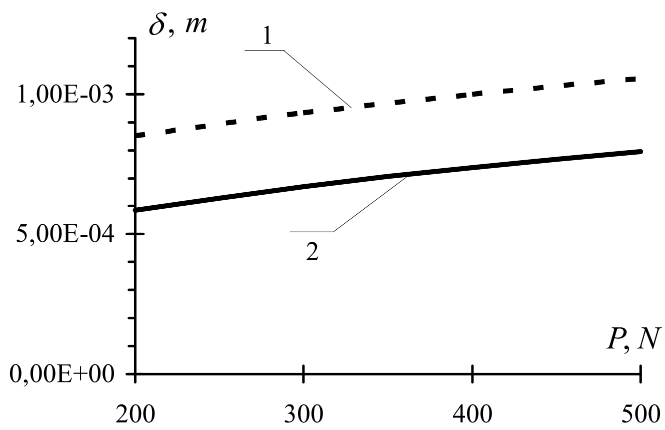

Figure 4. Maximal contact displacement:

1 - smooth ball and smooth half-space;
2 - coated smooth ball and rough half-space
($E^* = 1{,}5 \cdot 10^{11}\, N/m^2$, $R = 0{,}1\, m$, $H_r = 2{,}0 \cdot 10^{-5}$m, $L = 0{,}5 \cdot 10^{-3}\, m$, $\sigma_y = 12{,}0 \cdot 10^7$ N/m^2, $\lambda = 2$, $br = 2$) (Alexseev N.M., 1973; Kravchuk et al., 2000);

10 Conclusions

In this paper, the nonlocal model is applied to the contact of bodies with a circular area of interaction. The external load is supposed to be constant. A solution is obtained with the help of the contracting mapping principle and a simple iteration method. It is established that the first analytical approximation of the solution of equation (17) is useful for practical applications with sufficient accuracy.

It is established that the maximal contact displacement for the interaction of coated and rough bodies is less than the same parameters of smooth bodies without coating.

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