

Optimal Design of Stiffened Cylindrical Shells Based on an Asymptotic Approach

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The linear differential equations describing the free vibrations of ring-stiffened thin cylindrical shells are solved with the help of asymptotic techniques. The received approximate formulas are used for the evaluation of optimal parameters corresponding to the maximal fundamental frequency of the ring-stiffened shell with given mass.

1 Introduction

The fundamental vibration frequency is an important characteristic of a thin-walled structure. A simple way to raise the fundamental frequency to avoid resonance is to increase the thickness of the structure. However in this case the mass of the structure also increases. An optimal design of thin-walled structure lets to raise its frequency without the increase of its mass. The optimal design of a stiffened shell is a rather complicated problem. The method based on an asymptotic approach permits creating simple algorithms for the calculation of the optimal parameters. The results obtained in Filippov (1997, 1999) show that the replacement of a cylindrical shell by the optimal stiffened cylindrical shell with the same mass can increase the fundamental vibration frequency several times more.

In Filippov (1997, 1999) and almost in all studies of ring-stiffened shells, including Yang and Zhou (1995); Wang and Swaddiwudhipohg (1999), the rings have been considered as circular beams (beam model). Such traditional formulation of the problem permits estimating the optimal area of the ring cross-section, but does not permit to find the optimal form of the cross-section. If the cross-section is a rectangle, then the fundamental frequency increases monotonically with the increase in the ratio $k = b/a$, where b and a are the width and the thickness of the ring. However, for large values of k , the ring must be considered as a thin plate (plate model). The evaluation of the fundamental frequency of the ring-stiffened shell with the help of asymptotic method for large values of k is presented by Filippov and Haseganu (2003).

In this paper the beam and plate models are used for the approximate calculation of the optimal values of the parameter for the shell and ring with rectangular cross-section. An algorithm for the evaluation of the optimal parameters corresponding to the maximum value of the fundamental vibration frequency of the ring-stiffened shell with a given mass is developed. In particular, the optimal values of the parameter k are obtained.

2 Two Models

To compare two models of a ring we consider the vibrations of a thin cylindrical shell stiffened at one edge by a ring with rectangular cross-section. We take the radius R , of the cylindrical shell as a characteristic length, introduce the local coordinates $s \in [0, l]$, $\varphi \in [0, 2\pi]$ on the middle surface of the shell and denote u , v , w the components of the displacement (see Figure 1).

After the separation of variables

$$u(s, \varphi) = u(s) \cos m\varphi \quad v(s, \varphi) = v(s) \sin m\varphi \quad w(s, \varphi) = w(s) \cos m\varphi \quad (1)$$

the non-dimensional equations describing the free vibration of the cylindrical shell can be written in the following

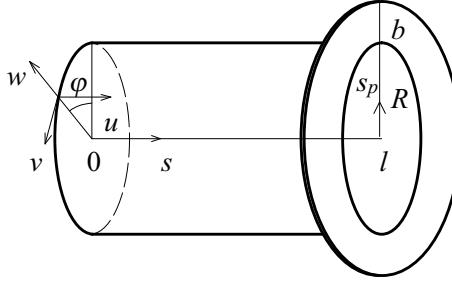


Figure 1. Cylindrical shell stiffened by ring

form

$$\begin{aligned} T'_1 + mS + \lambda u &= 0 & S' - mT_2 + Q_2 + 2H' + \lambda v &= 0 & Q'_1 + mQ_2 - T_2 + \lambda w &= 0 \\ Q_1 &= M'_1 + 2mH & Q_2 &= -mM_2 & M_1 &= h^2(\vartheta'_1 + \nu m\vartheta_2)/12 \\ M_2 &= h^2(m\vartheta_2 + \nu\vartheta'_1)/12 & H &= h^2(1 - \nu)\vartheta'_2/12 & T_1 &= u' + \nu(w + mv) \\ T_2 &= w + mv + \nu u' & 2S &= (1 - \nu)(v' - mu) & \vartheta_1 &= -w' & \vartheta_2 &= mw + v \end{aligned} \quad (2)$$

where $(')$ denotes the derivative with respect to the axial coordinate s , $\lambda = 4\pi^2\sigma\rho f^2R^2E^{-1}$ is the frequency parameter, $\sigma = 1 - \nu^2$, ν is Poisson's ratio, E is Young's modulus, ρ is the mass density, f is the vibration frequency, $T_1, T_2, S, Q_1, Q_2, M_1, M_2, H$ are the dimensionless stress-resultants and stress-couples, ϑ_1 and ϑ_2 are the angles of rotation of the normal, h is the dimensionless shell thickness.

Let the edge of the shell $s = 0$ is clamped, i.e.

$$u = v = w = \vartheta_1 = 0 \quad \text{for} \quad s = 0 \quad (3)$$

Assuming, that the thickness of the ring a is small and $k = b/a \sim 1$, where b is the width of the ring, we consider the ring as a circular beam and can use at the shell edge $s = l$ the approximate boundary conditions obtained in Filippov (1999):

$$mS - Q_1 + \sigma \frac{F}{h} m^2 (mv + w) = 0 \quad mQ_1 - S + \sigma \frac{J}{h} m^4 (mw + v) = 0 \quad T_1 = 0 \quad M_1 = 0 \quad (4)$$

where $F = ab$ and $J = ab^3/12$ are the area and the moment of inertia of the ring cross-section.

In this case for the evaluation of the dimensionless frequency parameter λ and the vibration modes we have the boundary value problem (2)–(4).

If the parameter k is large, we have to use another model for the ring and consider it as an annular thin plate. The non-dimensional equations describing the transverse flexural vibrations of the circular plate have the form

$$\begin{aligned} (s_p Q_{1p})' + mQ_{2p} + \lambda s_p w_p &= 0 & s_p Q_{1p} &= (s_p M_{1p})' - M_{2p} & s_p Q_{2p} &= -mM_{2p} + 2H_p \\ s_p M_{1p} &= a^2 [s_p \vartheta'_{1p} + \nu(m\vartheta_{2p} + \vartheta_{1p})]/12 & s_p M_{2p} &= a^2(m\vartheta_{2p} + \vartheta_{1p} + \nu s_p \vartheta'_{1p})/12 \\ H_p &= a^2 s_p (1 - \nu)\vartheta'_{2p}/12 & \vartheta_{1p} &= -w'_p & s_p \vartheta_{2p} &= mw_p \end{aligned} \quad (5)$$

Here $(')$ denotes the derivative with respect to the radial coordinate, $s_p \in [1, 1+b]$, w_p is the transverse displacement (deflection), $Q_{1p}, Q_{2p}, M_{1p}, M_{2p}, H_p$ are the dimensionless stress-resultants and stress-couples, ϑ_{1p} and ϑ_{2p} are the angles of rotation of the normal.

The tangential (in plane) vibrations of the plate are described by the following equations:

$$\begin{aligned} (s_p T_{1p})' - T_{2p} + mS_p + \lambda s_p u_p &= 0 & s_p S'_p + 2S_p - mT_{2p} + \lambda v_p &= 0 \\ s_p T_{1p} &= s_p u'_p + \nu(mv_p + u_p) & s_p T_{2p} &= u_p + mv_p + \nu s_p u'_p \\ 2s_p S_p &= (1 - \nu)(s_p v'_p - mu_p - v_p) \end{aligned} \quad (6)$$

where u_p and v_p are the tangential components of the displacement, T_{1p}, T_{2p}, S_p are the dimensionless stress-resultants.

At the circumference $s = l$, $s_p = 1$, the continuity conditions

$$\begin{aligned} w &= u_p & u &= -w_p & v &= v_p & \vartheta_1 &= \vartheta_{1p} \\ hQ_1 &= aT_{1p} & hT_1 &= -aQ_{1p} & hS &= aS_p & hM_1 &= aM_{1p} \end{aligned} \quad (7)$$

have to be satisfied.

We assume that the edge of the plate $s_p = 1 + b$ is free, and impose the boundary conditions

$$T_{1p} = Q_{1p} = S_p = M_{1p} = 0 \quad \text{for } s_p = 1 + b \quad (8)$$

To find λ using the plate model of the ring one has to solve equations (2), (5) and (6) taking into account the boundary condition (3), (7) and (8).

The boundary value problems for the plate and the beam models have been solved in Filippov and Haseganu (2003). In Figure 2 one can see the effect of k on the value of fundamental vibration frequency f in Hz. The parameters of the shell and the ring take on the following values: $a = h = 0.01$, $l = 2.5$, $b = ka$, $\nu = 0.3$, $R = 10$ in, $E = 3 \cdot 10^7$ psi, $\rho = 0.00073$ lb·s²/in⁴.

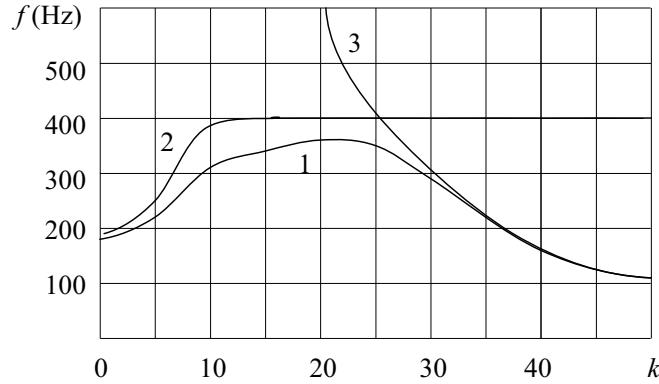


Figure 2. Fundamental frequency of the cylindrical shell stiffened by ring vs. $k = b/a$

Curve 1 represent the results of the numerical integration of the equations (2), (5) and (6) with the boundary condition (3), (7) and (8). Curve 2 plots approximate value f_b of fundamental frequency obtained by the solution of the boundary value problems (2)–(4). Curve 3 shows the first asymptotic approximation f_p according to the plate model which is near to the fundamental frequency of the plate with a clamped edge $s_p = 1$ and the free edge $s_p = 1 + b$.

The case $k = 0$ corresponds to the free shell edge $s = l$. For $k < 15$ the frequency f_b increases with k , because increases the stiffness of the beam. If $k > 15$ then f_b varies slowly since the stiffness of the beam is so large that it change do not have an essential influence on f_b . For such values of k the frequency f_b is near to the fundamental frequency 400 Hz of the cylindrical shell with freely supported edge $s = l$, corresponding to infinite stiffness.

For $k < 25$ the annular plate is narrow and its fundamental frequency is higher than f_b . Therefore, the fundamental frequency f is close to f_b . Further increasing k results in the decrease of the fundamental frequency f , approaching to f_p , since for $k < 25$ the plate is wide and its fundamental frequency is under the fundamental frequency f_b .

For small k both the shell and the plate vibrate while the circumferential wave number satisfies the inequality $m > 1$. For large k only the plate vibrates (the shell is practically motionless) and vibrations are axisymmetric ($m = 0$).

The numerical results show that the fundamental frequency of the cylindrical shell stiffened by the ring, $f(k)$, takes on the maximum value for some $k = k^*$. This optimal value k^* is not very much different from the value corresponding to the crossing point of curves 2 and 3 (see Figure 2). Therefore, the approximate value of k^* is the root of the equation

$$f_b(k) = f_p(k) \quad (9)$$

We use farther equation (9) to calculate the optimal parameters for the cylindrical shell stiffened by any number of rings, because it is easier to find f_b and f_p than exact value of the fundamental frequency.

3 Asymptotic Method for Analysis of Plate Model

The results presented in Filippov and Haseganu (2003) show that for large k the low-frequency vibrations of the cylindrical shell stiffened at the edge by the plate are axisymmetric ($m = 0$). In the case $m = 0$ the system of ordinary differential equations (2) splits into the system

$$\frac{h^2}{12} \frac{d^4 w}{ds^4} + \nu \frac{du}{ds} + w = \lambda w \quad \frac{d^2 u}{ds^2} + \nu \frac{dw}{ds} = -\lambda u \quad (10)$$

describing axisymmetric vibrations of the cylindrical shell and the equation

$$\frac{d^2 v}{ds^2} + 2(1 + \nu)\lambda v = 0 \quad (11)$$

describing torsional vibrations. An analogous splitting takes place for system (6), describing the tangential vibrations of the annular plate. Therefore, for $m = 0$ we obtain two separate boundary-value problems for axisymmetric and torsional vibrations. We consider only axisymmetric vibrations because the frequencies of torsional vibrations are higher than the frequencies of axisymmetric vibrations.

The equations, describing the axisymmetric vibrations of the annular plate, have the following form

$$\frac{a^2}{12} \Delta^2 w_p = \lambda w_p \quad \Delta w_p = \frac{1}{s_p} \frac{d}{ds_p} \left(s_p \frac{dw_p}{ds_p} \right) \quad (12)$$

$$s_p \frac{d^2 u_p}{ds_p^2} + \frac{du_p}{ds_p} - \frac{u_p}{s_p} = -\lambda u_p \quad (13)$$

Equations (12) describe the flexural vibrations of the plate, while equations (13) describe the vibrations in the plane of the plate.

The solutions of equations (10)–(13) satisfy the following boundary conditions

$$u = w = \vartheta_1 = 0 \quad \text{for} \quad s = 0 \quad (14)$$

$$\begin{aligned} w &= u_p & u &= -w_p & \vartheta_1 &= \vartheta_{1p} & hM_1 &= aM_{1p} \\ hQ_1 &= aT_p & hT_1 &= -aQ_{1p} & \text{for} & & s = l & s_p = 1 \end{aligned} \quad (15)$$

$$T_{1p} = M_{1p} = Q_{1p} = 0 \quad \text{for} \quad s_p = 1 + b \quad (16)$$

We assume that $a \ll 1$ and $h \ll 1$ and use for the solution the boundary-value problem (10)–(16) the asymptotic method depicted in Filippov and Haseganu (2003). Let seek the solutions of system (10) in the form

$$w = w_0 + w_1 + w_2 \quad u = u_0 + u_1 + u_2 \quad (17)$$

Here w_0 and u_0 are the solutions of the momentless system

$$\nu \frac{du}{ds} + w = \lambda w \quad \frac{d^2 u}{ds^2} + \nu \frac{dw}{ds} = -\lambda u \quad (18)$$

The functions

$$\begin{aligned} w_1 &= h^\alpha \sum_{j=1}^2 C_j e^{r_j s/h^{1/2}} & w_2 &= h^\alpha \sum_{j=3}^4 C_j e^{r_j(s-l)/h^{1/2}} \\ u_1 &= -h^{\alpha+1/2} \nu \sum_{j=1}^2 \frac{C_j}{r_j} e^{r_j s/h^{1/2}} & u_2 &= -h^{\alpha+1/2} \nu \sum_{j=3}^4 \frac{C_j}{r_j} e^{r_j(s-l)/h^{1/2}} \end{aligned} \quad (19)$$

where C_j are the arbitrary constants,

$$r_j = g \exp \left(\frac{5\pi}{4} - \frac{\pi j}{2} \right) i \quad j = 1, 2, 3, 4 \quad i^2 = -1 \quad g = \sigma^{1/4} \quad (20)$$

describe the boundary effect. The functions u_1 and w_1 decrease rapidly away from the shell edge $s = 0$. The functions u_2 and w_2 are very small everywhere except near the edge $s = l$.

The low-frequency vibrations correspond to $\lambda \sim a^2$. The form of asymptotic solution depends on the ratio h/a . The case $h = a$ is considered in Filippov and Haseganu (2003). In this case $\alpha = 1$, and in the first approximation

$$u_0 = u_1 = w_0 = w_1 = u_p = 0 \quad (21)$$

Therefore, in the first approximation we obtain the boundary-value problem for equation (12) with the boundary conditions

$$w_p = \vartheta_{1p} = 0 \quad \text{for} \quad s_p = 1 \quad M_{1p} = Q_{1p} = 0 \quad \text{for} \quad s_p = 1 + b \quad (22)$$

corresponding to the clamped plate edge $s_p = 1$ and free edge $s_p = 1 + b$.

This result explains why for large k the fundamental frequency of the cylindrical shell stiffened by the ring approaches the fundamental frequency of the plate with the clamped edge $s_p = 1$.

The case $h^{5/2} \sim a^3$ is of great importance in asymptotic analysis. In this case formulae (21) remain valid, $\alpha = 1/2$, and for $s = l$, $s_p = 1$ in the first approximation $w_p = 0$,

$$w_2 = 0 \quad \vartheta_{1p} = \vartheta_1 = \frac{dw_2}{ds} \quad aM_{1p} = hM_1 = h^3 \frac{d^2w_2}{ds^2} \quad (23)$$

If we substitute (19) into (23), we get

$$C_3 + C_4 = 0 \quad \vartheta_{1p} = r_3 C_3 + r_4 C_4 \quad M_{1p} = h^{5/2} (r_3^2 C_3 + r_4^2 C_4) \quad (24)$$

It follows from (24) that

$$aM_{1p} = c_p \vartheta_{1p} \quad c_p = \sqrt{2}gh^{5/2} \quad (25)$$

This boundary condition corresponds to the elastic supported edge. Therefore, in case $h^{5/2} \sim a^3$ we have the following boundary conditions for equation (12):

$$w_p = 0 \quad aM_{1p} = c_p \vartheta_{1p} \quad \text{for} \quad s_p = 1 \quad M_{1p} = Q_{1p} = 0, \quad \text{for} \quad s_p = 1 + b \quad (26)$$

where $aM_{1p} \sim a^3$, $c_p \sim h^{5/2}$, $\vartheta_{1p} \sim 1$.

If $a^3 \ll h^{5/2}$ then conditions (26) have the form (22). In particular, $a^3 \ll h^{5/2}$ if $a = h$. If $a^3 \gg h^{5/2}$, then $M_{1p}(1) = 0$ and the two first conditions (26) transform into the boundary conditions, corresponding to the simply supported edge $s_p = 1$.

The solution of equation (12) can be written in the form

$$w_p(s_p) = C_1 I_0(\gamma s_p) + C_2 J_0(\gamma s_p) + C_3 K_0(\gamma s_p) + C_4 Y_0(\gamma s_p) \quad \gamma = \lambda^{1/4} \quad (27)$$

Here I_0 , J_0 , K_0 and Y_0 are Bessel functions. Substituting (27) into (26) permits us to determine the arbitrary constants C_k and the frequency parameter λ .

4 Approximate Methods for Analysis of Beam Model

Let us consider the low-frequency vibrations of a thin cylindrical shell. One can obtain an approximate solution for equations (2) by means of the asymptotic method expounded in Bauer et al. (1993). For the shell with simply supported edges the first approximation of asymptotic method yields

$$\lambda = \frac{\sigma\alpha^4}{m^4} + \frac{m^4 h^2}{12} \quad (28)$$

where α is the eigenvalue for the boundary value problem

$$\frac{d^4 w}{ds^4} - \alpha^4 w = 0 \quad (29)$$

$$w = \frac{d^2 w}{ds^2} = 0 \quad \text{for} \quad s = 0 \quad s = l \quad (30)$$

The boundary value problem (29) and (30) has the nonzero solutions $w = \sin \alpha_n s$, if $\alpha_n = \pi n / l$, $n = 1, 2, \dots$. The lowest frequency parameter λ_1 corresponds to $\alpha_1 = \pi / l$. Taking into account that the parameter $m \sim h^{-1/4}$ is large we obtain from (28) the approximate formula

$$\lambda_1 = 2\alpha_1^2 \sqrt{h\sigma/12} \quad (31)$$

Let us consider next a thin cylindrical shell with simply supported edges, stiffened at the parallels $s_j = jl/(n+1)$, $j = 1, 2, \dots, n$ by n identical rings of rectangular cross-section (see Figure 3, where $n = 5$). The rings have the thickness a and the width $b = ka$.

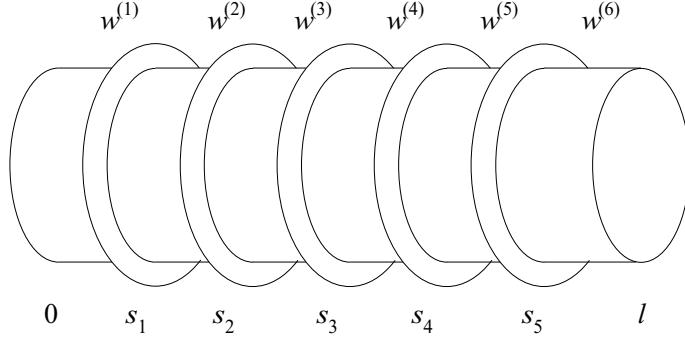


Figure 3. Ring-stiffened cylindrical shell

In this case the asymptotic method for the low-frequency vibrations in the first approximation gives $n+1$ equations

$$\frac{d^4 w^{(j)}}{dx^4} - \alpha^4 w^{(j)} = 0 \quad j = 1, 2, \dots, n+1 \quad (32)$$

The function $w^{(j)}$ is the normal deflection of the shell part lying between the rings or between a ring and the shell edge.

If the parameter k is not large we can use the beam model for the rings. We assume that the rings and the shell are made of the same material and $\max(a, b) \sim h^{3/4}$. Then the boundary conditions for equations (32) on the parallels $s = s_j$, derived in Filippov (1999), can be written in the form

$$w^{(j+1)} = w^{(j)} \quad \frac{dw^{(j+1)}}{ds} = \frac{dw^{(j)}}{ds} \quad \frac{d^2 w^{(j+1)}}{ds^2} = \frac{d^2 w^{(j)}}{ds^2} \quad \frac{d^3 w^{(j+1)}}{ds^3} = \frac{d^3 w^{(j)}}{ds^3} + cw^{(j)} \quad (33)$$

where $c = a^4 k^3 m^8 / (12h)$.

The boundary conditions on the shell edges $s = 0$ and $s = l$ have the following form:

$$w^{(1)} = \frac{d^2 w^{(1)}}{ds^2} = 0 \quad \text{for} \quad s = 0 \quad w^{(n+1)} = \frac{d^2 w^{(n+1)}}{ds^2} = 0 \quad \text{for} \quad s = l \quad (34)$$

The eigenvalues α of the boundary value problem (32)–(34) satisfy the equation $G(\alpha) = 0$, where $G(\alpha)$ is a determinant of order $2n$ with elements depending on the parameters m and c .

We can find the least values of λ from the formula

$$\lambda_i(c) = \min_m \left(\frac{\sigma \alpha_i^4(m, c)}{m^4} + \frac{h^2 m^4}{12} \right) \quad (35)$$

where $\alpha_i(m, c)$ are the roots of the equation $G(\alpha) = 0$, $i = 1, 2, \dots$. The case $c = 0$ corresponds to the non-stiffened shell. In this case $\alpha_i = \pi i / l_0$, and one can use (31) for the evaluation of $\lambda_1(0)$.

Some of the roots α_i , namely the roots $\alpha_q^* = q\pi(n+1)/l$, $q = 1, 2, \dots$, satisfy $G(\alpha) = 0$ for any $c > 0$. As follows from (35),

$$\lambda^* = \lambda_1(0)(n+1)^2 \quad (36)$$

is the lowest frequency parameter independent on c .

Let $\lambda_1(c)$ denote the smallest of the $\lambda_i(c)$. The function $\lambda_1(c)$ increases while inequality $\lambda_1(c) < \lambda^*$ holds. The value c^* for which

$$\lambda_1(c^*) = \lambda^* \quad (37)$$

is called the effective stiffness of the ring. For $c > c^*$ we have the equality $\lambda_1(c) = \lambda^*$.

It is possible to apply the Rayleigh's method for the approximate evaluation of the eigenvalue α_1 . The Rayleigh's formula may be written in a dimensionless form:

$$\alpha_1^4 = (I_1 + I_2)/I_0 \quad I_1 = \int_0^l \left(\frac{d^2 w}{ds^2} \right)^2 ds \quad I_2 = c \sum_{i=1}^{n-1} w^2(s_i) \quad I_0 = \int_0^l w^2 ds \quad (38)$$

Substituting into (38) the first vibration mode of the non-stiffened shell, $w_1 = \sin(\pi s/l)$, and taking into account the formula

$$\sum_{i=1}^{n-1} \sin^2(\pi i/n) = n/2,$$

we obtain

$$\alpha_1^4 = \left(\frac{\pi}{l} \right)^4 + \frac{c(n+1)}{l} \quad (39)$$

Hence,

$$\lambda_1(\eta) \simeq \lambda_1(0)(1+\eta)^{1/2} \quad \eta = \kappa c \quad \kappa = \frac{12c\sigma(n+1)}{lh^2m^8} \quad (40)$$

Formula (40) is valid for all η such that $\eta \leq \eta^* = \kappa c^*$. The formulae (37) and (40) yield an approximate expression for the effective stiffness:

$$\eta^* \simeq (n+1)^4 - 1 \quad (41)$$

It follows from (36) and (40) that

$$\frac{\lambda_1(\eta)}{\lambda_1(0)} \simeq \begin{cases} (1+\eta)^{1/2} & 0 \leq \eta \leq \eta^* \\ (n+1)^2 & \eta > \eta^* \end{cases} \quad (42)$$

From (42) we deduce that the fundamental vibration frequency increases with the reinforcement of the shell. The stiffened shell has the mass $M_s = M_0 + M_b$, where M_0 is the mass of non-stiffened shell, and M_b is the mass of the rings. Therefore, the stiffened shell is heavier than the non-stiffened shell. It is more interesting to compare the vibration frequencies for the stiffened and non-stiffened shells of equal mass.

If the non-stiffened cylindrical shell has the thickness h_0 , then its mass is $M_0 = M(h_0) = 2\pi R^3 h_0 \rho l$. We assume that the stiffened shell has a thickness $h < h_0$ and a mass $M_s = M(h) + M_b = M_0$, where $M_b = 2\pi R^3 \rho n a^2 k$ is the mass of the rings.

Using (42) we get

$$r_b^2 = \frac{f_b^2}{f_0^2} \simeq \begin{cases} d(1 + \eta)^{1/2} & 0 \leq \eta \leq \eta^* \\ d(n + 1)^2 & \eta > \eta^* \end{cases} \quad (43)$$

where f_b and

$$f_0 = \frac{1}{2\pi R} \sqrt{\frac{E\lambda_1(0)}{\sigma\rho}} \quad (44)$$

are the fundamental frequencies of the stiffened and non-stiffened shell respectively and $d = h/h_0$. The values of the parameters of the stiffened shell for which the function r_b attains its maximum r_b^* are called optimal values. For sufficiently small h_0 we obtain $r_b^* = (n + 1)\sqrt{d^*}$, where d^* is a root of the cubic equation

$$d^3 - q(d - 1)^2 = 0 \quad q = \sigma k l (n + 1) / (h_0 n^2 \eta^*) \quad (45)$$

The optimal values of d and a are d^* and $a^* = \sqrt{(1 - d^*)h_0 l / nk}$.

The dependence of r_b^* on n and k for a shell with parameters $l_0 = 4$, $h_0 = 0.01$ and $\nu = 0.3$ is shown in Figure 4.

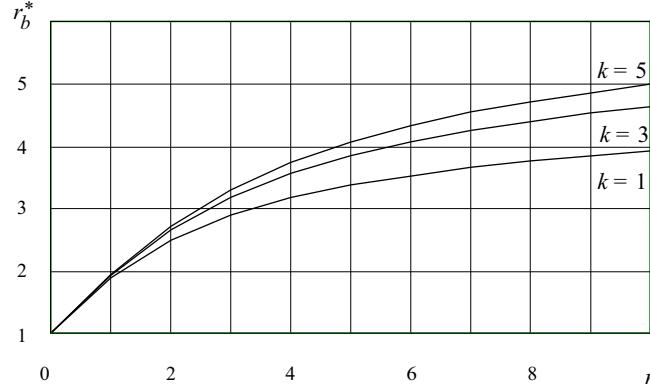


Figure 4. The ratio $r_b^* = f_b^* / f_0$ vs. $k = b/a$ and number of rings n

The function r_b^* increases with n and k . We can not use the asymptotic method for large n . For large k instead of the beam model we must use the plate model.

5 Evaluation of the Optimal k

To estimate the optimal value $k = k^*$ for the thin cylindrical shell with simply supported edges, stiffened by n identical rings we use equation (9) in the form

$$r_b(k) = f_b/f_0 = f_p/f_0 = r_p(k) \quad (46)$$

Here f_b and f_0 can be found from (43) and (44). To obtain f_p we solve the boundary-value problem (12), (26). Since every plate is joint with the two cylindrical shells we replace the constant c_p in boundary conditions (26) by constant $2c_p$.

At first we choose some value $k = k_0$ and find r_b^* , a^* and $b^* = ka^*$. Then, for the plate with the parameters $a = a^*$ and $b = b^*$, we solve the boundary-value problem (12), (26) and estimate r_p . If $r_b^* > r_p$ ($r_b^* < r_p$), we choose some $k < k_0$ ($k > k_0$) and repeat the same procedure until the equality $r_b^* = r_p$ is fulfilled with the necessary precision. Thus, instead of the set of curves in Figure 4, corresponding to different k , we obtain the curve (see Figure 5), corresponding to the more exact solution $r^* = f^*/f_0$ and the optimal $k = k^*$.

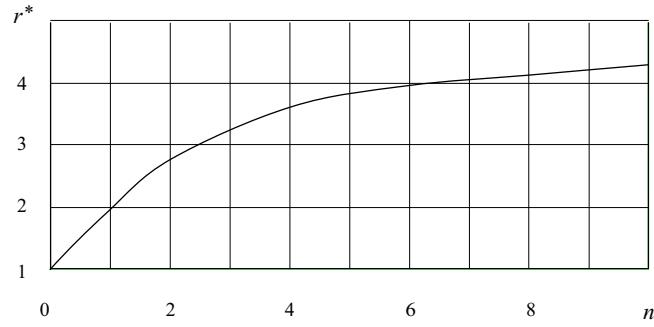


Figure 5. The ratio $r^* = f^*/f_0$ vs. number of rings n

The values of the optimal parameters of the stiffened shell for different numbers of rings, n , are given in the Table 1, where h^* is the optimal thickness of the stiffened shell (the thickness of the non-stiffened shell of equal mass

n	h^*	a^*	k^*	$1 + b^*$	f^*/f_0
1	0.00979	0.00439	43.6	1.192	1.98
2	0.00905	0.00894	23.9	1.213	2.85
4	0.00528	0.0362	3.61	1.131	3.63
6	0.00316	0.0446	2.29	1.102	3.93
8	0.00213	0.0442	2.01	1.089	4.15
10	0.00154	0.0424	1.88	1.080	4.32

Table 1. The optimal parameters

is $h_0 = 0.01$), a^* is the optimal thickness of the ring, $1 + b^*$ is the optimal outer radius of the plate (the inner radius is equal to 1), f^* is the fundamental frequency of the stiffened shell with the optimal parameters, f_0 is the fundamental frequency of the non-stiffened shell.

The thickness h^* decreases as the number n increases. For $n < 6$ the increase in n causes a rapid increase in a^* and f^*/f_0 and a rapid decrease in k^* . For $n > 6$ the function $a^*(n)$, $f^*/f_0(n)$ and $k^*(n)$ vary slightly.

6 Conclusions

In the current paper the rings of the ring-stiffened shell have been considered as beams and as annular thin plates. As a consequence, the problem becomes more complicated in comparison with problems analyzed in Filippov (1999). However, the new approach permits obtaining more exact and realistic solutions by means of asymptotic integration methods. It was shown that the replacement of a non-stiffened cylindrical shell by the optimal stiffened cylindrical shell with the same mass can increase the fundamental frequency of a structure more than four times.

Acknowledgment

This work supported by RFBR, grant 04-01-00257.

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