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The Multiple Field Concept of Structural Analyses with Applications to Static and Dynamic Shape Control

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Dedicated to the memory of Professor Dr. Dr.h.c.mult Friedrich P.J. Rimrott

Interpreting any imposed strain or strain rate, e.g., the thermal strain, as a source in the isothermal background structure, renders what is nowadays called the multiple field analysis. Such imposed strains (strain rates), refer also to viscoplastic deformation (with ductile damage taken into account), to render time-variant distributed sources in the linear elastic background. A direct multiple field analysis of uniaxial elastic-plastic waves is presented along these lines including the transient plastic zone behind a plastic shock-front. Even in a three-dimensional setting, the waves sent out by an instantaneous localized plastic source are identified for a fixed receiver station by a time convolution. Applications of these dynamic multiple fields seem to be promising when monitoring elastic-plastic structures which are subjected to overloads. In vibration problems, the superposition principle remains incrementally applicable, despite of the actual material nonlinearity and the consequently resulting space-time field coupling. The solution is split into the quasistatic one (no inertia is taken into account) and the dynamic one (with homogeneous dynamic boundary conditions). The dynamic solution is consequently approximated by a low order modal expansion. The analyses are either based on the differential equations of motion with the linear operator of the background structure on the left hand side and the forcing terms including the distributed sources on the right hand side or on reciprocity theorems, e.g., in the form of the generalized Maysel's formula well known from linear thermoelasticity. The intensity of the plastic sources is sufficiently well approximated by the strain increments resulting from constitutive relations where the plastic strain rate is given in terms of the current state. An important application refers to interface slip of vibrating layered (slender) structures, like composite beams, plates or shells. Such a multiple field formulation is also extremely well suited to tackle linear quasi-static or dynamic deformation control problems by distributed actuators. Controlled heating of members in frames or trusses of space structures for slow processes (without additional thermal stresses) or use of the piezoelectric effect in composite 'thin-walled" structures like beams, plates and shells also working under rapidly changing loading conditions, all are considered by the integral equation approach. Actual or virtual use of the Green's stress dyadic of the background (possibly with relaxed boundary conditions) is understood.

1 Introduction

F.P.J. Rimrott's tremendous research efforts as documented in nearly hundred papers in refereed journals span from elasto-viscoplastic stress and lifetime problems, Rimrott and Marin (1958) and Rimrott (1959, 1992), through deployable space structures, Rimrott (1965), including static and dynamic thermal stress problems, Rimrott (1971), to composite structures, Rimrott and Muensterer (1971). Rimrott (1994) presented a famous article on shells. An attempt is made in this paper, to memorize this part of his research achievements. For this sake, the multiple field concept is adopted. Rimrott's great success in gyromechanics and in space mechanics cannot be accounted for in this context.

Classically, a background is considered where coupled fields actuate sources and thus new processes develop. In solid mechanics, linear thermoelasticity is the best explored problem setting, when the imposed thermal strain is considered a source distributed in the isothermal elastic background: thus, rendering the work done by a unit force on the thermoelastic displacement equal to the work of the complementary Green's stress dyadic done on the thermal strains. That generalized reciprocity relation is called Maysel's formula, see Maysel (1941), and, e.g., Ziegler and Irschik (1987) for its derivation from the principle of virtual displacements and for several aspects of its structural applications. In Irschik et al. (1993), the dynamic generalization is worked out with the time convolution of the complementary Green's stress dyadic derived from a unit instantaneous force in the background, with the transient strain source taken into account. As a side product, the Mura-Willis integral of infinite bodies, see Mura (1991), was made applicable to finite dimensional structures with prescribed displacement boundary conditions on part of the surface, while the remaining part is kept free of traction.

Such a dynamic version of Maysel's formula was successfully applied to the one-dimensional multiple fields arising in an impacted ductile semi-infinite rod. Superposition of the elastic waves in the background rod that are produced by the given (severe) impact load and (taking into account the causality in the Mach-plane) by the sources of non-compatible plastic strain increments (with distribution and intensity given by the constitutive equation of the actual viscoplastic material) actuated within the transient plastic zone, solve the nonlinear problem in terms of the multiple coupled fields and further render a stable numerical algorithm, reference Irschik and Ziegler (1990). Ductile damage and adiabatic depositing of the dissipated heat was included in Irschik and Ziegler (1995) without complicating the computation. Multiple field analyses of elastic-plastic waves were extended to the three-dimensional problem of point symmetric spherical waves emanating from the explosively loaded surface of a cavity. In reference Fotiu and Ziegler (1996), the dynamic version of Maysel's formula is applied, in Ziegler et al. (1995), the analysis is based on the non-homogeneous D'Alembert wave equation of the displacement potential. Effects of adiabatically depositing the dissipated heat, i.e., considering the transient thermal stresses at the wave front, plastic damage, etc., are taken into account. Further, dramatic pulse changes are observed in the overtaking process of unloading wave fronts. The extension to three-dimensional problems (without spherical symmetry) is still in preparation. A first result is derived in Ziegler (2001), where the waves emitted from a concentrated plastic source, say actuated by overload, in a ductile layered plate, and received at a surface point are considered. The latter problem is basic in the course of monitoring ductile structures with respect to the onset of plastic deformation. Since plane interfaces are considered, complementary Green's stress dyadic in the background is consequently expanded into plane waves, see Pao and Gajewski (1977) and Ziegler and Borejko (2000), for the generalized ray integrals emitted from point sources.

For standing waves, i.e., vibration problems, we refer in this connection to the dynamic processes in linearized thermoelasticity which have been explored in detail and the methods of analysis are well documented in text books and in the international journals of mechanics. Most importantly, the superposition principle is applicable and the solution is consequently split into the quasistatic one (no inertia is taken into account) and the dynamic one. In such a case the dynamic boundary conditions become homogeneous and the remaining dynamic solution is approximated by a low order modal expansion. The linear elastic thermal shock problem of a beam has been analyzed along these lines by Boley and Barber (1957). The extension of this classical paper to include plastic deformations is worked out in Irschik and Ziegler (1985). Rimrott (1981) discussed the most important frequency criterion for thermally induced vibrations in elastic beams. Nonlinear vibrations of yielding structures, analyzed by means of the generalized Maysel's formula and separation of the response, i.e., consideration of the plastic drift, are discussed in Irschik and Ziegler (1988). In the course of the development of plastic zones during the transient phase of forced vibrations of a beam the excitation of higher order modes of the elastic background has been observed and evaluated in reference Fotiu et al. (1990). High sensitivity of the structural response with respect to the onset of yielding at the upper or lower fiber is observed, indicating what may be called "plastic chaos", see Moon (1987) for the latter. Ductile damage based on micro-mechanical considerations has been incorporated in the multiple field analysis in Fotiu et al. (1991). Such a smeared ductile damage process couples the elastic and viscoplastic strain fields, see Dafalias (1977). Plates, experiencing alternating plasticity under various load conditions are considered along these lines in Fotiu (1992) and Fotiu (1993).

Control of the linear elastic load deformation by imposed eigen-strains (of thermal or piezoelectric origin) remains a linear problem when using the background concept and Maysel's formula, see Irschik et al. (1998) for control of the deflection of a composite beam. Such a direct approach, saving the solution of the nonlinear field optimization, was worked out in terms of compatible imposed strain fields without stress by Irschik and Ziegler (2001), with applications presented in Irschik et al. (2000a). Control of deformations and stresses in thermally loaded bodies, with an optimization procedure applied, is given by Nyashin et al. (2000). To account for the dynamic problem of vibration annihilation, it was shown, Irschik and Pichler (2001), that any time parametrized static solution suffices to suppress the vibrations when forced by time- and space- separable loads. The evaluations in Irschik and Pichler (2003) validated these results and made the cancellation of the dynamic load stress portion evident. Cancellation of small vibrations with respect to a moving frame is analyzed in Irschik et al. (2003) and in Irschik et al. (2004). Three proceedings of recent IUTAM-Symposia summarize the application of smart materials, edited by Bahei-El-Din and Dvorak (1998), Gabbert and Tzou (2001) and Watanabe and Ziegler (2003), see also Irschik (2002).

Within the limits of linear elasticity, the multiple field approach, when properly interpreted yields, for beams, plates and shells, higher order solutions in terms of the classical (lower order) theory, see again Irschik and Ziegler (1988) and for shear deformable plates, Irschik et al. (2000b). For composite structures see Irschik and Ziegler (1998). These aspects of the background concept are not further discussed in this paper.

The stable numerical scheme common to dynamic and static problems results by discretizing the fields of imposed strains, say by assigning constant intensities of the strain source in small volumes, i.e., in cells. For constitutive relations where the nonlinear strain rate is given as a function of the current state (even with the dependence on internal variables taken into account) first order increments resulting from stepping the time are commonly found sufficiently well fitting approximations in the course of updating the intensities of the imposed sources. Hence, any (computing-time consuming) iteration loops are saved.

2 Reciprocity Theorems and Generalized Maysel's Formula

A reciprocity theorem of the theory of elasticity provides an integral relation between two states of the same linearly elastic body. Betti's principal theorem of elastostatics relates two loading states, labeled A and B, body forces f_i, surface traction t_i, displacements denoted u_i, Betti (1872). The same relation holds true for two distinct time-harmonic states with common assigned frequency, even for two bodies with perfect interface S^{in}, where, e.g., domain $V = V_1 \cup V_2$ and surface $S = S_1 \cup S_2$, Achenbach et al. (1982),

$$\int_V \left(f_i^A u_i^B - f_i^B u_i^A \right) \mathrm{dV} + \oint_S \left(t_i^A u_i^B - t_i^B u_i^A \right) \mathrm{dS} = 0 \tag{2.1}$$

The *Laplace* transformed general dynamic reciprocity takes on the same form, however, the transformed body forces are generalized to include nonhomogeneous initial conditions, ρ is the density,

$$\overline{g_i(\xi; s)} = \bar{f}_i + \rho \left[s\, u_i\left(0^+\right) + \dot{u}_i\left(0^+\right) \right], \qquad t \geq 0 \tag{2.2}$$

de Hoop (1995). Achenbach (2000) made extensive use of dynamic reciprocity to calculate efficiently elastic wave fields.

Nowacki (1975), extended the dynamic reciprocity in frequency space to include distributed sources of self-stress $\varepsilon_{k,l}^*$ acting in the possibly anisotropic linear elastic background with generalized Hooke's law,

$$\sigma_{ij} = c_{ijkl} \left(\varepsilon_{kl} - \varepsilon_{kl}^* \right), \qquad i, j, k, l = 1, 2, 3, \qquad c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij} \tag{2.3}$$

to

$$\int_V \left(f_i^A u_i^B - f_i^B u_i^A \right) \mathrm{dV} + \int_V c_{ijkl} \left(\varepsilon_{kl}^{*A} \varepsilon_{ij}^B - \varepsilon_{kl}^{*B} \varepsilon_{ij}^A \right) \mathrm{dV} = 0 \tag{2.4}$$

assuming one and the same boundary conditions in states A and B, namely prescribed displacements on part of the surface with the remaining part free of traction.

Equation (2.4) is specified in a problem oriented manner. State A is identified with the distributed source loading of the background, hence, $f_i^A = 0$. Case B is taken as the auxiliary dummy force problem of the background, assuming the solution known, thus $\varepsilon_{kl}^{*B} = 0$. Considering the symmetries of the elasticity tensor, Eq. (2.3) and substituting Hooke's law of state B,

$$\sigma_{kl}^B = c_{klij} \varepsilon_{ij}^B \tag{2.5}$$

transform the kernel of the second volume integral in Eq. (2.4) to

$$c_{ijkl} \varepsilon_{kl}^{*A} \varepsilon_{ij}^B = c_{klij} \varepsilon_{ij}^B \varepsilon_{kl}^{*A} = \sigma_{kl}^B \varepsilon_{kl}^{*A} \tag{2.6}$$

The result in the *Laplace* transform domain (with time convolution apparent, and for homogeneous initial conditions) is the dynamic generalization of Maysel's formula of thermoelasticity,

$$\int_V \overline{f_i^B}\, \overline{u_i^A} \,\mathrm{dV} = \int_V \overline{\varepsilon_{kl}^{*A}}\, \overline{\sigma_{kl}^B} \,\mathrm{dV} \tag{2.7}$$

Considering the dummy body force distribution in state B instantaneous (thus rendering the dynamic stress dyadic σ_{kl}^B in the background) yields

$$W^{(B,A)}(t) = \int_V F_i^B(\xi) u_i^A(\xi,t) \mathrm{dV}(\xi) = \int_0^t \int_V \varepsilon_{kl}^{*A}(\xi,\tau) \sigma_{kl}^B(\xi,t-\tau) \mathrm{dV}(\xi) \mathrm{d}\tau \tag{2.8}$$

The volume integral, with all the advantages inherent, is interpreted as the virtual work done by instantaneous body forces F_i^B on the source induced displacements u_i^A . Equation (2.8), e.g., is applied to piezoelectrically induced vibrations of thin, layered shells of revolution, Irschik and Ziegler (1996).

3 Nonlinear Structural Vibrations – Generalized Maysel's Formula

3.1 Maysel's Formula and the Static Green's Function

Considering current states at time t and $t + \Delta t$, applying D'Alembert's principle, see, e.g., (Washizu 1975, p.393), twice, i.e., the principle of virtual displacements with inertial forces taken into account, and taking the difference, yields, see Nagtegaal (1982) for the basis of nonlinear FEM,

$$\int_V (\Delta f_i - \rho \Delta \ddot{u}_i) \delta \Delta u_i \, \mathrm{dV} + \oint_S \Delta t_i \, \delta \Delta u_i \, \mathrm{dS}$$
$$- \int_V \left\{ \sigma_{ij} \, \delta \left(\frac{1}{2} \Delta u_{l,i} \, \Delta u_{l,j} \right) + \Delta \sigma_{ij} \left[\frac{1}{2} \left(\delta \Delta u_{j,i} + \delta \Delta u_{i,j} \right) + \left(\frac{1}{2} u_{l,j} \, \delta \Delta u_{l,i} + u_{l,i} \, \delta \Delta u_{l,j} \right) \right] \right\} \mathrm{dV} = 0$$

When choosing the static Green's displacements of the auxiliary dummy force problem for virtual displacements, and $\widetilde{u}_{i(k)}$, boundary conditions can be properly relaxed, we derive at the intermediate result used by Mukherjee and Chandra (1984), as the starting point of nonlinear BEM,

$$0 = \int_V (\Delta f_i - \rho \Delta \ddot{u}_i) \widetilde{u}_{i(k)} \, \mathrm{dV} + \oint_S \Delta t_i \, \widetilde{u}_{i(k)} \, \mathrm{dS}$$
$$- \int_V \left\{ \Delta \sigma_{ij} \left[\widetilde{\varepsilon}_{ij(k)} + \left(\frac{1}{2} u_{l,j} \, \widetilde{u}_{l,i(k)} + u_{l,i} \, \widetilde{u}_{l,j(k)} \right) \right] + \sigma_{ij} \, \frac{1}{2} \left(\Delta u_{l,i} \, \widetilde{u}_{l,j(k)} + \Delta u_{l,j} \, \widetilde{u}_{l,i(k)} \right) \right\} \mathrm{dV} \tag{3.1}$$

Substituting the principle of virtual displacements of the auxiliary dummy unit force problem of the background,

$$1.\Delta u_k + \int_S \widetilde{X}_{i(k)} \, \Delta u_i \, \mathrm{dS} - \int_V \widetilde{\sigma}_{ij(k)} \, \Delta \varepsilon_{ij} \, \mathrm{dV} = 0 \tag{3.2}$$

together with Hooke's law in its incremental form of Eq. (2.3) and Hooke's law of the auxiliary force problem, Eq. (2.5) as well, render a natural separation of the various load contributions, Irschik and Ziegler (1988),

$$1 \, \Delta u_k = \Delta u_k^{0(S)} + \Delta u_k^{*(S)} + \Delta u_k^{(D)} + \Delta u_k^{nl.geom.} \tag{3.3}$$

where the quasistatic response to the given external load is

$$\Delta u_k^{0(S)} = \int_S \Delta t_i^{(n)} \, \widetilde{u}_{i(k)} \, \mathrm{dS} + \int_V \Delta f_i \, \widetilde{u}_{i(k)} \, \mathrm{dV} \tag{3.4}$$

and the quasistatic response to sources of eigenstress, the natural definition of the plastic drift increment becomes

$$\Delta u_k^{*(S)} = \int_V \widetilde{\sigma}_{ij(k)} \, \overline{\Delta \varepsilon_{ij}} \, \mathrm{dV} \tag{3.5}$$

The correction term of the nonlinear geometric relations, $\Delta u_k^{nl.geom.}$, depends on the stress and displacement increments and on the current total values of strain and displacements as well. Its contribution is considered in the engineering approximations in detailed applications. The dynamic increment of the displacement is defined by the integral equation,

$$\Delta u_k^{(D)} = -\int_V \rho \, \Delta \ddot{u}_i \, \tilde{u}_{i(k)} \, dV \tag{3.6}$$

which becomes linear for linearized geometric relations. Splitting the increment according to the two load cases, analogous to the quasistatic increment, Eqs. (3.4) and (3.5),

$$\Delta u_k^{(D)} = \Delta u_k^{0(D)} + \Delta u_k^{*(D)} \tag{3.7}$$

yields two integral equations, superscripts ^{0} and * are respectively understood in Eq. (3.8),

$$\Delta u_k^{(D)} + \int_V \rho \, \Delta \ddot{u}_i^{(D)} \, \tilde{u}_{i(k)} \, dV = -\int_V \rho \, \Delta \ddot{u}_i^{(S)} \, \tilde{u}_{i(k)} \, dV \tag{3.8}$$

Since time is a parameter in the quasistatic response, a proper time spline function may be substituted in the separable quasistatic displacement increment, and the twofold parameter differentiation is analytically performed. A linear ramp function, e.g., renders two impacts at the beginning and at the end of the time interval. In special cases such a procedure proves even efficient for the approximation of the external load increment. Modal expansion of the complementary dynamic increment is recommended for frequency band limited excitations assuring accelerated convergence. With respect to the determination of stress states, the quasistatic part eventually contains singularities in the Green's functions which can be easily accounted for, thus rendering an overall stable numerical routine.

3.2 Flexural Vibrations of Ductile Beams, Homogeneous Cross Section

Considering Eq. (2.3) for the uniaxial stress state in the slender beam,

$$\Delta \varepsilon - \overline{\Delta \varepsilon} = E^{-1} \Delta \sigma \tag{3.9}$$

kinematic hardening and ductile damage and a yield surface,

$$|\sigma - \eta| = (1 - D)\sigma_Y , \qquad \Delta \sigma = \Delta \eta - \sigma_Y \Delta D , \qquad \Delta \eta = \overline{E}(1 - D)\Delta \varepsilon^p \tag{3.10}$$

yield the increment of the imposed eigenstress source in the plastic zone, see, e.g., Fotiu et al. (1987),

$$\overline{\Delta \varepsilon} = (1 - D - \Delta D)\Delta \varepsilon^p + (D + \Delta D)\Delta \varepsilon + (\varepsilon - \varepsilon^p)\Delta D \tag{3.11}$$

Considering the incremental dissipated work,

$$\Delta W_D = (1 - D)^{-1} \sigma_Y \left[1 - \left(1 + E / \overline{E}\right)^{-1}\right] |\Delta \varepsilon| \tag{3.12}$$

exponential growth of the damage coefficient below its critical value is a plausible assumption, Fotiu et al. (1990),

$$D = D_{cr} \left[1 - \exp\left(-\alpha W_D^2\right)\right] , \qquad \alpha > 0 \tag{3.13}$$

The integral equations (3.8) are solved for the deflection by substituting the modal expansions, $\phi_n(x)$ are the natural modes of the background beam,

$$w^{0,*(D)} = \sum_{n=1}^{N^{0,*}} Y_n^{0,*}(t)\phi_n(x) \tag{3.14}$$

with the result, light viscous modal damping has been added at this stage,

$$\Delta\ddot{Y}_n^{0,*} + 2\varsigma_n\,\omega_n\,\Delta\dot{Y}_n^{0,*} + \omega_n^2\,\Delta Y_n^{0,*} = -L_n^{0,*}\,\ddot{f}^{0,*}/m_n\,\omega_n^2 \tag{3.15}$$

In case of the lateral load increment $\Delta P(x,t) = p(x)\,f^0(t)$, the classical participation factor results,

$$L_n^0 = \int_0^l \phi_n(x)\,p(x)\,\mathrm{dx} \tag{3.16}$$

The sources of eigenstress produce the quasistatic deflection, $\Delta w^{*(S)}(x;t) = \Delta w^{*(S)}(x)\,f^*(t)$, and, however, render the nonclassical participation factor,

$$L_n^* = \omega_n^2 \int_0^l \mu\,\phi_n(x)\,\Delta w^{*(S)}(x)\,\mathrm{dx} \tag{3.17}$$

The quasistatic deflection $w^{0(S)}$ can be determined by any common and efficient method, the incremental contribution of the plastic sources is best described by Maysel's formula, *z* is an axis of symmetry of the cross-section, area is *A*, and $\overline{m}$ denotes the imposed curvature,

$$\Delta w^{*(S)}(x) = \int_0^l \widetilde{M}(\xi;x)\,\overline{\Delta m}(\xi)\,\mathrm{d}\xi, \qquad \overline{\Delta m}(\xi) = \frac{1}{J_y}\int_A z\,\overline{\Delta\varepsilon}(\xi;z)\,\mathrm{dA} \tag{3.18}$$

A CC-beam with a uniformly distributed time harmonic load, $P(x,t) = p\,f^0(t)$, of reduced assigned forcing frequency, $\omega/\omega_1 = 3/2$, $f^0(t) = H(t)\sin\omega t$, i.e., switched on at time zero, serves an illustrative example and was considered in detail in Fotiu et al. (1990), with the following parameters taken into account

$$\zeta_n = 0.02,\ E/\sigma_Y = 2000,\ \bar{E}/E = 1/9,\ l/h = 20,\ \alpha\,\sigma_Y^2 = 50,\ D_{cr} = 0.95,\ pl^3/EJ_y = 1\,.$$

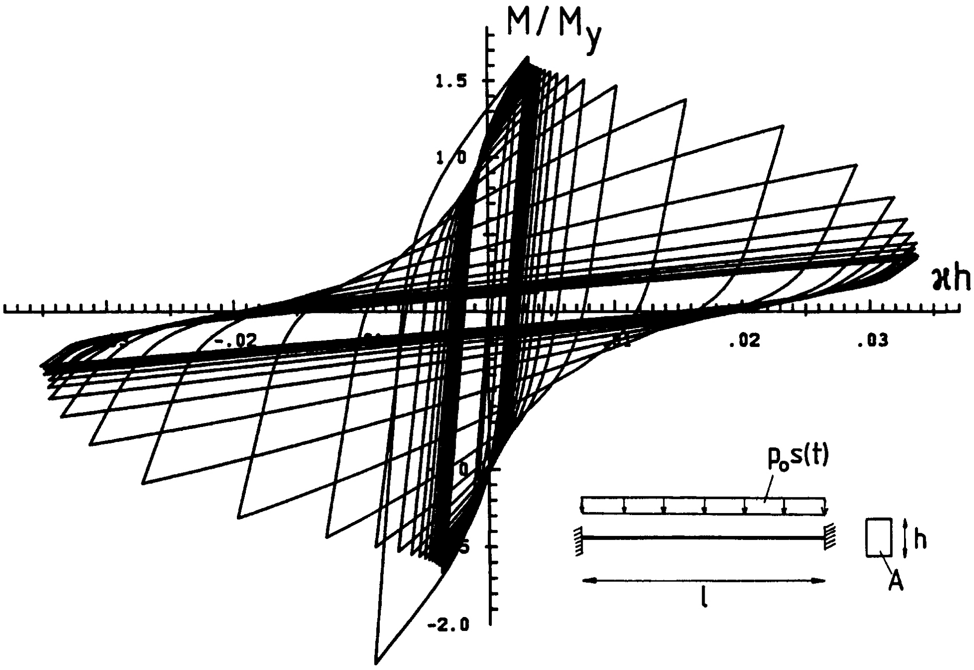

Figure 1: Nondimensional edge moment of a ductile CC-beam versus nondimensional curvature. Successive degradation due to ductile damage in cyclic plasticity illustrated. Fotiu et al. (1990).

In Figure 1, the edge-moment-curvature diagram is shown together with a sketch of the CC-beam. In Figure 2, the enrichment of the spectrum over the unlimited elastic one is indicated.

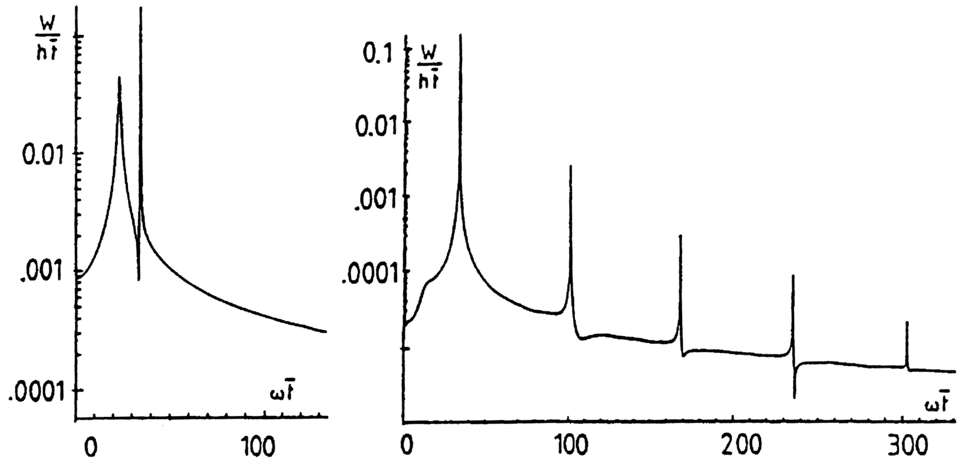

Figure 2: Nondimensional spectrum of the mid-span deflection. Forcing frequency is 50% above the basic linear frequency. Left: unlimited elastic. Right: plastic zone has developed. Fotiu et al. (1990).

Geometric nonlinearity is approximately considered by definition of a curvature

$$\kappa(\xi;t) = -\frac{N(t)}{EJ_y} w(\xi;t), \qquad N \approx \frac{EA}{l}\int_0^l \left[\frac{1}{2}\left(w_{,x}\right)^2 - \bar{n}\right]\mathrm{dx}, \qquad \bar{n} = \frac{1}{A}\int_{\mathrm{A}} \bar{\varepsilon}\,\mathrm{dA} \tag{3.19}$$

and its substitution as an additional source in Eq. (3.18),

$$\Delta w^{*(S)}(x;t) = \int_0^l \widetilde{M}(\xi;x)\left[\overline{\Delta m}(\xi;t) + \Delta\kappa(\xi;t)\right]\mathrm{d}\xi$$

The complementary moment influence function remains unaffected,

$$\widetilde{M}(\xi;x) = -EJ_y\,\widetilde{w}_{,\xi\xi}(\xi;x) \tag{3.20}$$

Oblique vibrations, i.e., plasticity under multiple dynamic load, are considered in Adam and Ziegler (1997a).

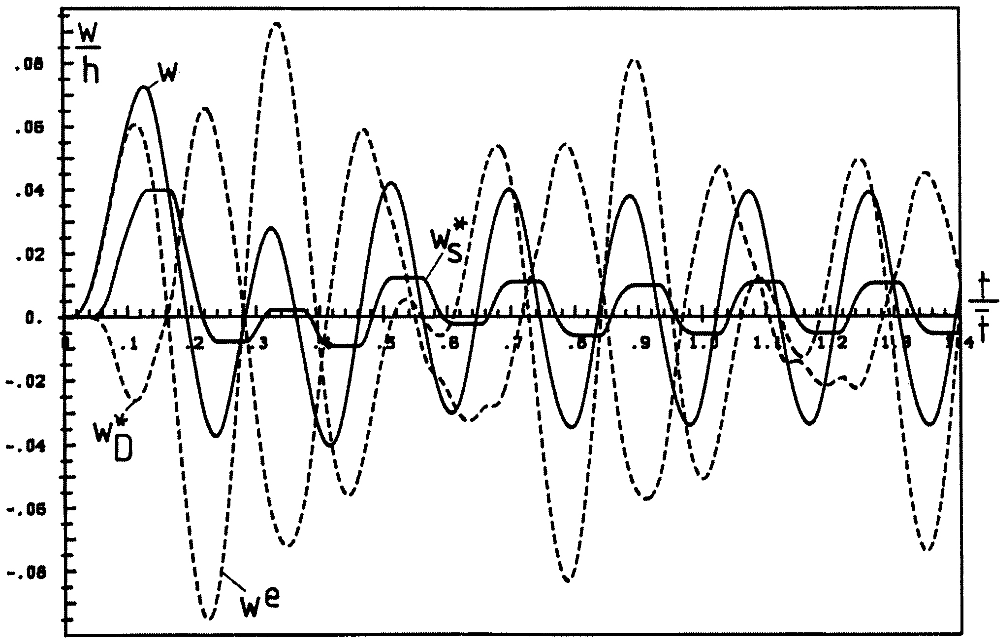

Figure 3: Time evolution of the mid-span deflection. Note the plastic drift w_S^* and the unlimited elastic response w^e. Fotiu et al. (1990).

3.3 Flexural Vibrations of Ductile Composite Beams

3.3.1 Symmetrical Layering

In Adam and Ziegler (1997b), thick elastic-plastic layers in a symmetrical arrangement are considered. Each layer is approximated as a Timoshenko beam perfectly bonded to its neighbor. Introducing an effective cross sectional rotation, Heuer (1992), and the effective curvature imposed by the sources of eigenstress and shear angle, yield moment and shear force analogous to a homogeneous beam with effective stiffness, result for three layers, a core and two equal faces with perfect bond, see Fig. 4,

$$M = \sum_{i=1}^{3} M_i = EJ_e\left(\psi_{e,x} - \bar{\kappa}_e\right), \qquad Q = \sum_{i=1}^{3} Q_i = S_e\left(\psi_e + w_{,x} - \bar{\gamma}_{me}\right) \tag{3.21}$$

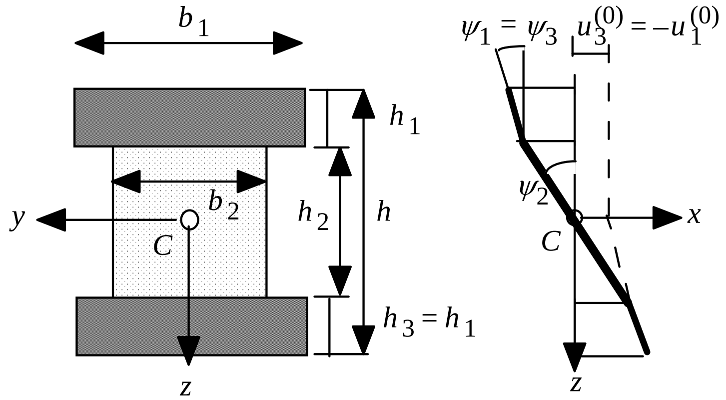

Figure 4: Cross section of symmetrically layered beam. Individual rotations of layer cross sections shown together with axial horizontal displacement $u^{(0)}$. Adam and Ziegler (1997b).

Accordingly a set of two second order equations of motion result, rotational inertia is neglected,

$$\begin{aligned} &-EJ_e\,\psi_{e,xx} + S_e\left(\psi_e + w_{,x}\right) = m(x,t) - EJ_e\,\bar{\kappa}_{e,x} + S_e\,\bar{\gamma}_{me} \\ &\mu_e\,\ddot{w} - S_e\left(\psi_{e,x} + w_{,xx}\right) = p(x,t) - S_e\,\bar{\gamma}_{me,x} \end{aligned} \tag{3.22}$$

The following classical boundary conditions are specified in analogy to the homogeneous shear-deformable beam,

(i) SS: $\quad w = 0, \qquad M \equiv EJ_e\left(\psi_{e,x} - \bar{\kappa}_e\right) = 0$

(ii) Free: $\quad M \equiv EJ_e\left(\psi_{e,x} - \bar{\kappa}_e\right) = 0, \qquad Q \equiv S_e\left(\psi_e + w_{,x} - \bar{\gamma}_{me}\right) = 0 \qquad (3.23)$

(iii) Core clamped (faces free): $\quad w = 0, \qquad \psi_2 = 0$

Note, the rigidly built-in boundary condition, $\psi_1 = \psi_2 = \psi_3 = 0$, is not within the scope of the effective beam theory.

Common deflection w and the effective cross-sectional rotation ψ_e are split according to the actions of the external load and the sources of eigenstress and further separated in the quasistatic and dynamic portions, see again Eq. (3.3), however, when reduced for linearized geometric relations. Source loading produces the increments,

$$\Delta w^{*(S)}(x;t)=\int_0^l\left[\widetilde{M}^{F(S)}(\xi;x)\Delta\kappa_e(\xi;t)+\widetilde{Q}^{F(S)}(\xi;x)\overline{\Delta\gamma_{me}}(\xi;t)\right]\mathrm{d}\xi \tag{3.24}$$

$$\Delta\psi_e^{*(S)}(x;t)=\int_0^l\left[\widetilde{M}^{M(S)}(\xi;x)\Delta\kappa_e(\xi;t)+\widetilde{Q}^{M(S)}(\xi;x)\overline{\Delta\gamma_{me}}(\xi;t)\right]\mathrm{d}\xi \tag{3.25}$$

with static influence functions given in textbooks, like that of Rubin and Vogel (1993). The dynamic increments of effective deformations are given in terms of the complementary dynamic Green's functions (of accelerated convergence) with linear ramp time functions accounted for, the contribution of the nonhomogeneous initial conditions are not delineated here, see Adam and Ziegler (1997b) again,

$$\Delta w^{*(D)}(x)=\int_0^l\left[\widetilde{M}^{F(D)}(\xi,\Delta t;x)\Delta\kappa_e(\xi)+\widetilde{Q}^{F(D)}(\xi,\Delta t;x)\overline{\Delta\gamma_{me}}(\xi)\right]\mathrm{d}\xi+\Delta w^{*(D)}\left(x|t_0\right) \tag{3.26}$$

$$\Delta\psi_e^{*(D)}(x)=\int_0^l\left[\widetilde{M}^{M(D)}(\xi,\Delta t;x)\Delta\kappa_e(\xi)+\widetilde{Q}^{M(D)}(\xi,\Delta t;x)\overline{\Delta\gamma_{me}}(\xi)\right]\mathrm{d}\xi+\Delta\psi_e^{*(D)}\left(x|t_0\right) \tag{3.27}$$

$$\widetilde{w}^{F,*(D)}(\xi,x,\Delta t)=-\mu_e^{-1}\sum_{n=1}^{N}\phi_n(\xi)\phi_n(x)\left(\omega_n^2\,\Delta t\right)^{-1}h_n(\Delta t)$$

$$\widetilde{\psi}_e^{F,*(D)}(\xi,x,\Delta t)=-\mu_e^{-1}\sum_{n=1}^{N}\psi_n(\xi)\phi_n(x)\left(\omega_n^2\,\Delta t\right)^{-1}h_n(\Delta t)$$

$$\widetilde{w}^{M,*(D)}(\xi,x,\Delta t)=-\mu_e^{-1}\sum_{n=1}^{N}\phi_n(\xi)\psi_n(x)\left(\omega_n^2\,\Delta t\right)^{-1}h_n(\Delta t)$$

$$\widetilde{\psi}_e^{F,*(D)}(\xi,x,\Delta t)=-\mu_e^{-1}\sum_{n=1}^{N}\psi_n(\xi)\psi_n(x)\left(\omega_n^2\,\Delta t\right)^{-1}h_n(\Delta t)$$

where the nth undamped unit impulse response function enters,

$$h_n(\Delta t)=\omega_n^{-1}\sin(\omega_n\,\Delta t)$$

In case of the SS-beam (i), Eq. (3.23), the ortho-normalized mode shapes are simply given by trigonometric functions,

$$\begin{aligned}&\phi_n=\left(\sqrt{l/2}\right)^{-1}\sin(\alpha_n x),\qquad \psi_n=-\left(\beta_n\sqrt{l/2}\right)^{-1}\alpha_n\cos(\alpha_n x),\qquad \alpha_n=n\pi/l\\&\omega_n=\alpha_n^2\sqrt{EJ_e/(\mu_e\,\beta_n)},\qquad \beta_n=1+\alpha_n^2\,EJ_e/S_e\end{aligned} \tag{3.28}$$

Considering the faces made of ductile aluminium, kept in distance by an elastic polyvinyl chloride core, all three layers of same thickness, and the lateral load switched on at time zero,

$$p(x,t)=q_0\sin(\pi x/l)H(t)\sin(\omega t),\qquad \omega/\omega_1=1.58,\qquad T_1=2\pi/\omega_1 \tag{3.29}$$

yields for slender beams, $\ell/h=10$, results which are in good agreement with those derived by the simpler equivalent-single-layer approximation. For high beams, $\ell/h=4$, results are shown in Figs. (5) and (6), much improved with respect to the simpler theory, see Adam and Ziegler (1997b) for more details. Note also the evolution of the axial displacement of the lower fiber in Fig. 7.

For inelastic interface slip (interface is considered physical as a thin layer) see Adam et al. (1999) for the symmetric composite and Adam and Ziegler (1998) for a sandwich beam with asymmetric layers.

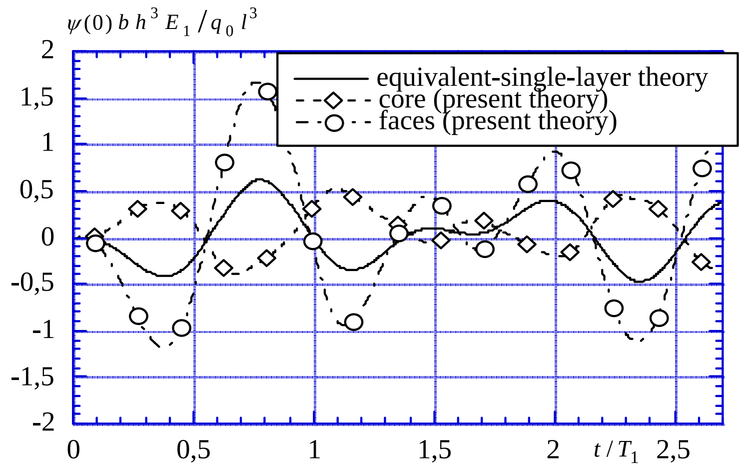

Figure 5: Individual cross-sectional rotations in single layer theory compared to the rough approximation of the equivalent-single-layer approximation, $l / h = 4$. Adam and Ziegler (1997b).

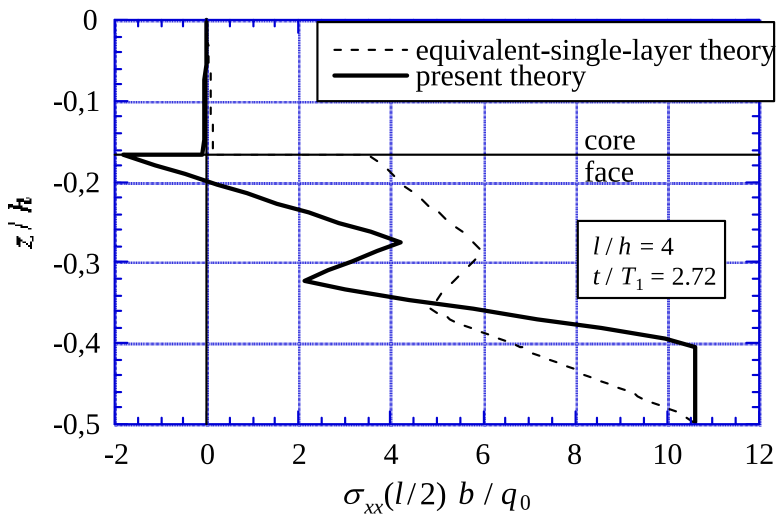

Figure 6: Nondimensional bending stress distribution in lower half of cross-section, unloading effect in face exhibited. Adam and Ziegler (1997b).

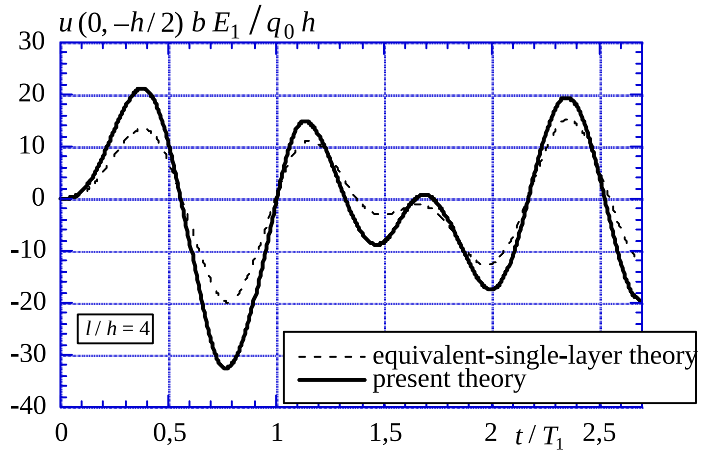

Figure 7: Nondimensional axial displacement of the tip of the lower fiber. Adam and Ziegler (1997b).

3.3.2. Nonsymmetrical Layering. Interface Slip

Adam et al. (1997) treated forced flexural vibrations of a two-layered beam with elastic interface bonding. In Adam et al. (2000), the formulation was extended to the case of thermal shock loading. The interlaminar shear force, see Fig. 8 for the stress-resultants, is set proportional to the jump in axial displacement at the interface,

$$T = k\,\Delta u \tag{3.30}$$

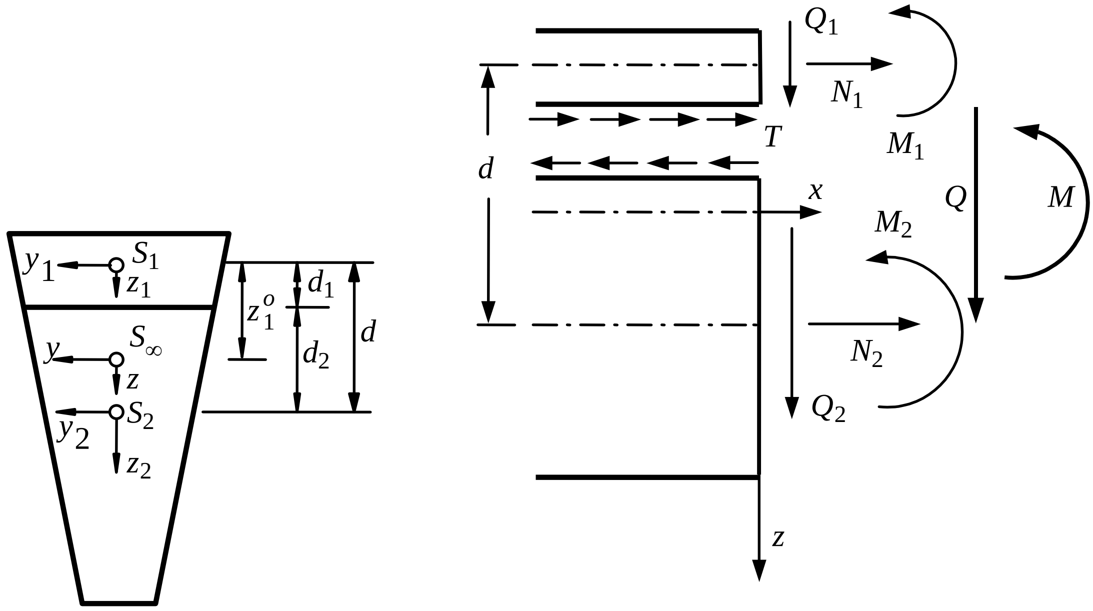

Figure 8: Two layer cross-section with elastic interface. Stress resultants at positive cross-section. Adam et al. (2000).

For inelastic slip see Adam and Ziegler (1999). Single layer theory and a perfect bond determines the effective parameters of the isothermal background. After some manipulations, the equation of motion becomes the sixth order p.d.e. with forcing terms originating from thermal and slip sources,

$$w^{vi} - \lambda^2 w^{iv} + \frac{\mu}{EJ_0}\ddot{w}^{ii} - \lambda^2 \frac{\mu}{EJ_\infty}\ddot{w} = -\kappa^{(0)}_{\theta,xxxx} + \lambda^2 \kappa_{\theta,xx} \tag{3.31}$$

where the averaged and gross thermal curvatures are respectively defined, see again Fig. 8 for the origins of the coordinates,

$$\kappa^{(0)}_\theta = \frac{1}{EJ_0}\left(EJ_1\,\kappa_{\theta 1} + EJ_2\,\kappa_{\theta 2}\right), \quad \kappa_{\theta i} = \frac{1}{J_i}\int_{A_i} \alpha\,\theta\, z_i \,\mathrm{dA}, \qquad \kappa_\theta = \frac{1}{EJ_\infty}\int_{A_0} \alpha\,\theta\, z\,\mathrm{dA} \tag{3.32}$$

and the effective parameters are,

$$EA_0 = EA_1 + EA_2, \quad EJ_0 = EJ_1 + EJ_2, \quad EJ_i = E_i\,J_i, \quad \lambda^2 = k\left(\frac{EA_0}{EA_1\,EA_2} + \frac{d^2}{EJ_0}\right) \tag{3.33}$$

if, for sufficiently thin layers, the Bernoulli-Euler assumption is applied. The deflection $w(x,t)$ when partitioned into its quasistatic and complementary dynamic part,

$$w(x,t) = w^{(S)}(x;t) + w^{(D)}(x,t) \tag{3.34}$$

allows for a convergent modal expansion of the latter for all the relevant stress resultants, and thus becomes necessary for rendering a useful solution. In Adam et al. (2000), the integral representation,

$$w^{(S)}(x;t) = \int_0^l \widetilde{w}_1^{(S)}(x,\xi)\kappa_\theta(\xi,t)\mathrm{d}\xi + \int_0^l \widetilde{\mathrm{w}}_2^{(S)}(x,\xi)\kappa_\theta^{(0)}(\xi,t)\mathrm{d}\xi \tag{3.35}$$

has been applied, where $\widetilde{w}_i^{(S)}, i = 1, 2$ are the "thermal" Green's deflections at point x initiated by single unit curvatures $\widetilde{\kappa}_\theta$ and $\widetilde{\kappa}_\theta^{(0)}$, respectively applied at ξ .

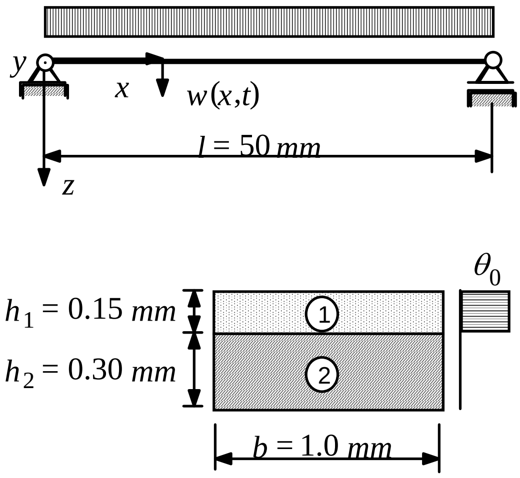

Figure 9: Sudden heating of the upper layer. Dimensions of SS-beam shown. Adam et al. (2000).

The complementary dynamic modal series exhibits accelerated convergence for deformations,

$$w^{(D)}(x,t) = \sum_{n=1}^{N} Y_n^{(D)}(t)\phi_n(x) \tag{3.36}$$

For the example of an SS-beam, as shown in Fig. 9, it was found in Adam et al. (2000) convenient and efficient to expand the quasistatic part analogously, rendering the modal amplitudes,

$$Y_n^{(S)}(t) = \omega_n^{-2} \int_0^l \phi_n(x)\left[\lambda^2 \kappa_{\theta,\mathrm{xx}}(x,t) - \kappa_{\theta,xxxx}^{(0)}(x,t)\right]\mathrm{dx} \tag{3.37}$$

and, consequently, light viscous damping is added at this stage, the set of modal equations results,

$$\ddot{Y}_n^{(D)} + 2\varsigma_n \omega_n \dot{Y}_n^{(D)} + \omega_n^2 Y_n^{(D)} = -\ddot{Y}_n^{(S)} - 2\varsigma_n \omega_n \dot{Y}_n^{(S)} \tag{3.38}$$

which are to be solved by Duhamel's convolution integral with nonhomogeneous initial conditions taken into account. The outcome of the numerical analysis, detailed in Adam et al. (2000), are summarized in the Figs. 10.1 and 10.2. Note the extreme cases of perfect and no bond at the interface and the intermediate cases given by two values of the elastic stiffness at the interface. Indirectly, the strong sensitivity of the response on the physical property of the interface is illustrated. The results of a convergence study shown in Figs. 11.1 and 11.2 impressively illustrate the merits of the proposed study.

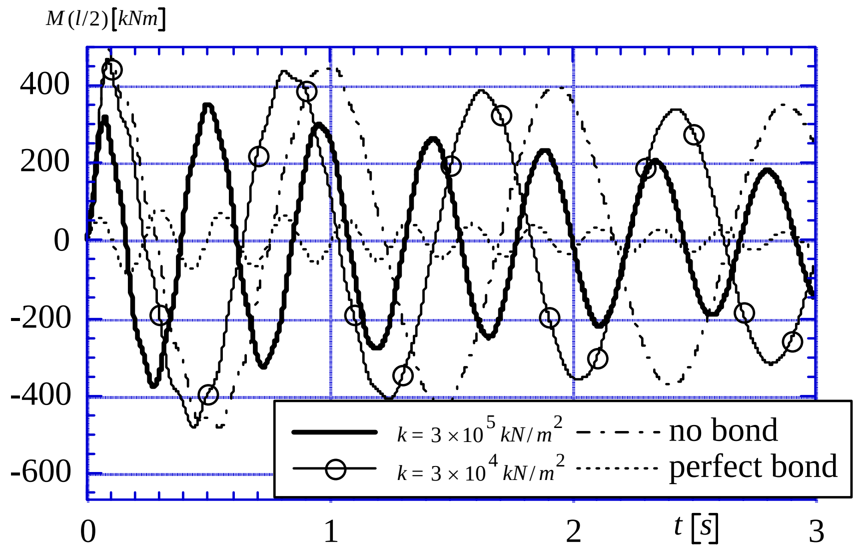

Figure 10.1: Evolution of bending moment at midspan. Adam et al. (2000).

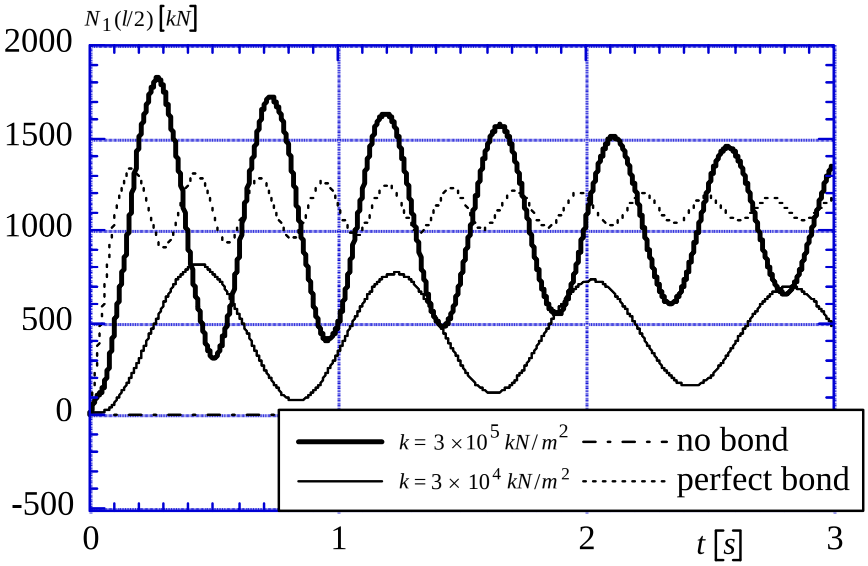

Figure 10.2: Evolution of normal force in layer #1 at midspan. Adam et al. (2000).

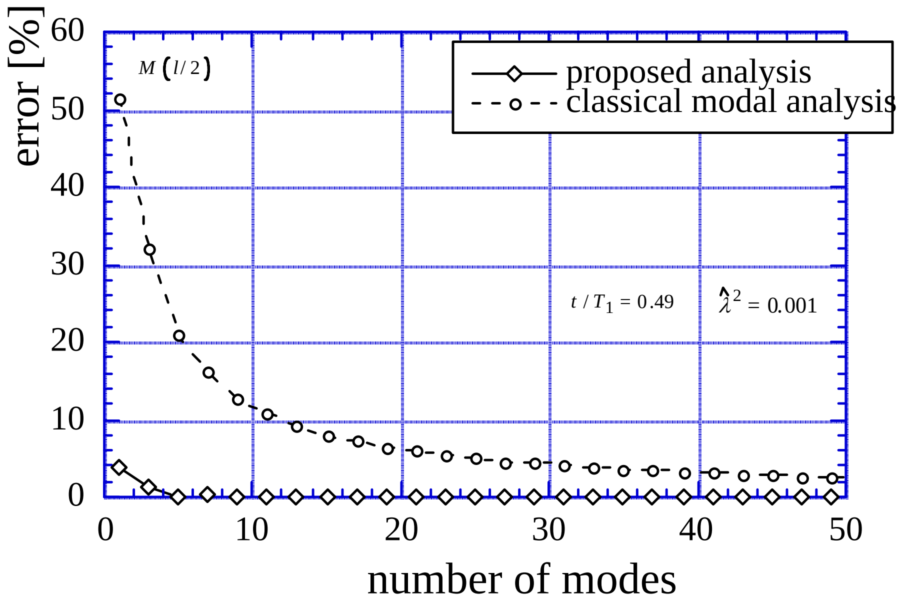

Figure 11.1: Fast convergence of complementary dynamic modal series of moment. Adam et al. (2000).

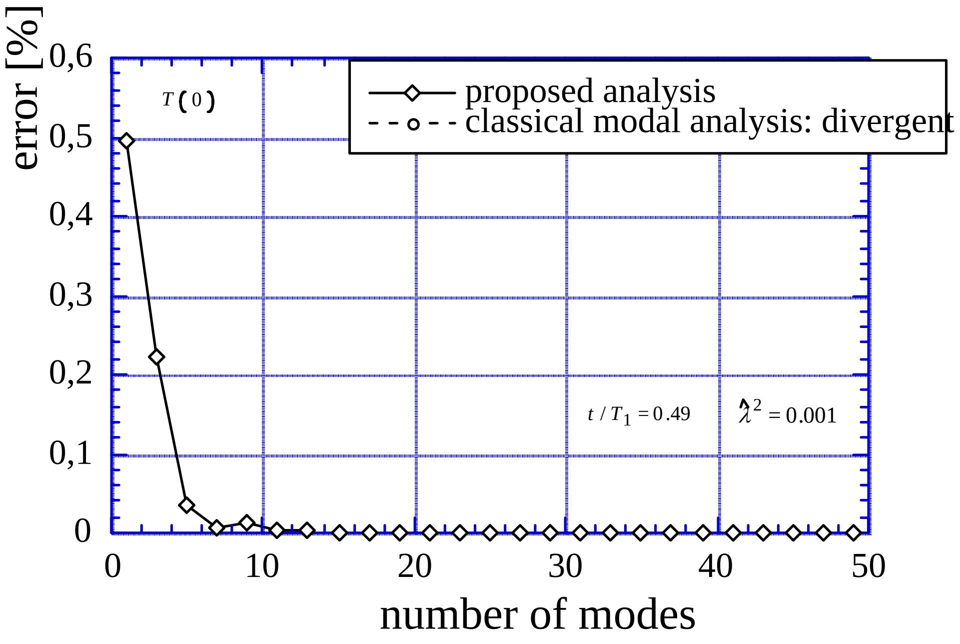

Figure 11.2: Convergence of complementary dynamic modal series of interfacial shear stress. Adam et al. (2000).

4 A Direct Approach to Shape Control of Linear Elastic Structures

Control of deformations became a major research effort. For recent reviews, see Crawley (1994), and for use of the piezoelectric effect, Rao and Sunar (1994) and Saravanos and Heyliger (1999). Composite shells are treated by Tzou (1993). A direct approach to control load-determined deformations of linear elastic structures has been developed in Irschik and Ziegler (2001), thus, circumventing the nonlinear optimization process. Extension to dynamic shape control is performed by Irschik and Pichler (2001). A short account based on Maysel's formula, Irschik and Ziegler (2001), is given below.

4.1 Static Shape Control by Eigenstrain without Stress

At first we consider the boundary value problem of a force loaded linear elastic structure in equilibrium. Its solution is presented in such a form that makes it possible, to identify the fields of eigenstrains, which produce the same deformations without solving the inverse problem. Hence, in a second section, the boundary value problem of the structure, loaded by actuators that produce eigenstrains, is considered. Part of the surface is kept immoveable in both cases.

4.1.1 Force Load

We consider the general boundary value problem of a linear elastic, possibly anisotropic body loaded by body forces and/or surface traction. The local equilibrium requires

$$\mathrm{div}\,\underline{\sigma} + \underline{b} = \underline{0} \tag{4.1}$$

On part of the boundary, kinematic boundary conditions apply,

$$\Gamma_u : \underline{u} = \underline{0}, \ldots. \tag{4.2a}$$

on the remaining part, the traction are prescribed,

$$\Gamma_\sigma : \underline{\sigma}.\underline{n} = \underline{t}^{(n)} \tag{4.2b}$$

Within the validity of linearized geometric relations,

$$\varepsilon_{ij(F)} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4.3}$$

and Hooke's law,

$$\varepsilon_{ij(F)} = C_{ijlm}\,\sigma_{lm} \tag{4.4}$$

the solution of the force-displacements by means of the principle of virtual forces, see Ziegler (1998)-the Green's stress dyadic $\tilde{\sigma}_{ij(k)}$ produced by a unit single force applied in a direction *k* is assumed known, - is given in the form of the volume integral, a complementary Green's formula,

$$1.u_{k(F)}\left(\underline{x}\right) = \int_V \tilde{\sigma}_{ij(k)}\left(\underline{\xi}, \underline{x}\right)\varepsilon_{ij(F)}\left(\underline{\xi}\right)\mathrm{dV}\left(\underline{\xi}\right) \tag{4.5}$$

which represents a virtual work relation.

The representation of the solution (4.5) is chosen for the sake of simplicity for identifying impotent eigenstrains in section 4.1.2.

4.1.2 Identification of Impotent Eigenstrains

Since we are interested in eigenstrain distributions that do not produce stress $\sigma_{lm(\varepsilon)}$, the equilibrium condition (4.1), in absence of body forces has the trivial solution,

$$\sigma_{lm(\varepsilon)} = 0 \tag{4.6}$$

The kinematic boundary condition (4.2a) applies and in absence of prescribed surface traction Eq. (4.2b) becomes

$$\Gamma_\sigma : \underline{\underline{\sigma}}_{(\varepsilon)} . \underline{n} = \underline{0} \tag{4.7}$$

Linearized geometric relations apply as well,

$$\varepsilon_{ij(\varepsilon)} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)_{(\varepsilon)} \tag{4.8}$$

and the generalized Hooke's law

$$\varepsilon_{ij(\varepsilon)} - \bar{\varepsilon}_{ij} = C_{ijlm}\, \sigma_{lm(\varepsilon)} \tag{4.9}$$

reduces according to Eq. (4.6) to

$$\varepsilon_{ij(\varepsilon)} - \bar{\varepsilon}_{ij} = 0, \qquad \bar{\varepsilon}_{ij} = \bar{\varepsilon}^*_{ij} \tag{4.10}$$

that means, the total strain is equal to the imposed impotent eigenstrain, denoted $\bar{\varepsilon}^*_{ij}$, throughout the structure.

The deformations produced by any field of eigenstrains in a body subjected to the above mentioned boundary conditions are given by the volume integral, which is a virtual work expression with the force Green's stress dyadic, $\tilde{\sigma}_{ij(k)}$, understood to be known,

$$1.u_{k(\varepsilon)}(\underline{x}) = \int_V \tilde{\sigma}_{ij(k)}(\underline{\xi}, \underline{x})\bar{\varepsilon}_{ij}(\underline{\xi})\mathrm{dV}(\underline{\xi}) \tag{4.11}$$

Equation (4.11) is again recognized to be a generalization of Maysel's formula of linear isotropic thermoelasticity. For its derivation by means of the principle of virtual displacements and its workability, e.g., with respect to the derivation of Green's stress dyadic under relaxed kinematic boundary conditions, see Ziegler and Irschik (1987). Prescribing the left hand side, Eq. (4.11) becomes the integral equation of static shape control. It is easily recognized that the inverse problem associated to shape control is ill-posed, since any field of eigenstrains which produce stresses but no deformation, so called nilpotent eigenstrains, subsequently denoted $\bar{\varepsilon}^{**}_{ij}$, can be superposed to the solution of the specified Eq. (4.11).

However, if we require the force displacements of Eq. (4.5) to be equal to the eigenstrain produced displacements of Eq. (4.11), the eigenstrains in the latter equation can be identified equal to the load strains apparent in Eq. (4.5). Thus, the eigenstrains are a compatible field of strains, they are impotent and do not produce stresses, for a definition see Mura (1991). Hence, annihilation of the deformations by force loading is achieved by producing eigenstrains (which are the total strains, see again Eq. (4.10)) equal to the force produced strains with reversed sign. No additional stresses result in this case. For detailed derivations see Irschik and Ziegler (2001),

$$\bar{\varepsilon}^*_{ij}(\underline{x}) = -\varepsilon_{ij(F)}(\underline{x}) \tag{4.12}$$

For discrete or discretized structures it will be shown, that nilpotent portions of the imposed eigenstrain can be constructed to redistribute load stresses, however severely constrained by the equilibrium conditions of the superposed loading. In conclusion, the duality, expressed by Eqs. (4.5) and (4.11), is identified as the basis of static shape control. Irschik et al (1994) and Irschik et al. (1995) applied $-\varepsilon_{ij(f)}(\boldsymbol{x})$ as eigenstrain fields to smart beam type structures.

However, the non uniqueness has been noted in Irschik and Ziegler (2001) which reflects the underlying illposed (in the sense of Hadamard) inverse problem. There exists deformation-free solutions of Eq. (4.11), what has been called "nilpotent solutions", see Irschik et al. (1999a) for inappropriate shape functions in control of flexural vibrations. The latter is extensively discussed in Krommer and Irschik (1999), Irschik et al. (1999b) and Irschik et al. (1999c), for plates see Agrawal et al. (1994), Irschik et al. (1997a) and Heuer et al. (1997), for shells see Irschik and Ziegler (1996) and Irschik et al. (1997b).

4.2 Static Control of Deflections of Beams

A cantilevered beam with perfectly symmetrically attached piezoelectric layers of rectangular cross-section has been considered by Irschik et al. (1998), see also Irschik et al. (1997a) and Kalamkarov and Drozdov (1998). Vinson (1992), observed the analogy between the piezoelectric actuation and the thermal effect which makes it possible to utilize the vast amount of thermoelastic solutions to account for the strains imposed piezoelectrically. For an assemblage of *n* truss- and beam-type structures, properly homogenized, Eq. (4.11) becomes the sum of spanwise integrals,

$$1\,\delta_{k(\varepsilon)}(x)=\sum_{j=1}^{n}\int_0^{l_j}\left[\widetilde{M}_{(k)}(\xi,x)\bar{\kappa}(\xi)+\widetilde{N}_{(k)}(\xi,x)\bar{\varepsilon}(\xi)\right]\mathrm{d}\xi \tag{4.13}$$

where the cross-sectional mean values with averaged extensional -, *D*, and flexural stiffness, *B*, are

$$\bar{\varepsilon}=\frac{1}{D}\int_A E^*\,\overline{\varepsilon_{xx}}\,\mathrm{dA},\quad \bar{\kappa}=\frac{1}{B}\int_A E^*\,\overline{\varepsilon_{xx}}\,z\,\mathrm{dA},\quad D=\int_A E^*\,\mathrm{dA},\quad B=\int_A E^*\,z^2\,\mathrm{dA} \tag{4.14}$$

and E^* is the effective modulus in the uni-axial generalized Hooke's law, thermal effects neglected,

$$\sigma_{xx(\varepsilon)}=E^*\left(\varepsilon_{xx(\varepsilon)}-\overline{\varepsilon_{xx}}\right) \tag{4.15}$$

Equation (4.11) of the load produced deformation by cross-sectional integration takes on the form,

$$1\,\delta_{k(f)}(x)=\sum_{j=1}^{n}\int_0^{l_j}\left[\widetilde{M}_{(k)}(\xi,x)\left(M_{(f)}(\xi)B^{-1}(\xi)\right)+\widetilde{N}_{(k)}(\xi,x)\left(N_{(f)}(\xi)D^{-1}(\xi)\right)\right]\mathrm{d}\xi \tag{4.16}$$

The deformations expressed by Eqs. (4.13) and (4.16) become equal if the imposed piezoelectric curvature $\bar{\kappa}$ and the piezoelectric mean strain $\bar{\varepsilon}$ are respectively selected

$$\bar{\kappa}(x)=M_{(f)}B^{-1}=-\partial^2 w_{(f)}/\partial x^2,\qquad \bar{\varepsilon}(x)=N_{(f)}D^{-1}=\partial u_{(f)}/\partial x \tag{4.17}$$

Thus, for the cantilevered beam of length ℓ with constant flexural rigidity $B=const$, rigidly built-in at $x=0$, we desire the deflection under uniformly distributed lateral load $f=q_0$, as the deflection of the redundant CS-beam, i.e. virtually simply supported at $x=\ell$,

$$w_{(\varepsilon)}(x)=\hat{w}_{(f)}(x)=q_0\,\ell^4\,\xi^2\left(3-5\xi+2\xi^2\right)/\,48B,\qquad \xi=x/\ell \tag{4.18}$$

Hence, as a consequence, we select the piezoelectric curvature by substituting Eq. (4.18) into the first of Eq. (4.17) and, with proper actuation of the piezoelectric layers, note $M_{(\varepsilon)}(x)=0$, render the prescribed total deflection $\hat{w}_{(f)}(x)$.

4.3 Shape Control of Discretized or Discrete Structures

The Finite Element Method, FEM, is the common tool for discretizing continuous structures. A-priori discrete structures are, e.g., idealized trusses. In case of linear elasticity and linearized geometric relations, a constant stiffness matrix, $\underline{K}$, relates the external loads, $\underline{F}$, with the nodal displacements, $\underline{u}$. Thus, Hooke's law (4.4) of the load case for a discrete structure becomes,

$$\underline{F} = \underline{K}\, \underline{u}_F \tag{4.19}$$

The solution of Eq. (4.19) is to be done by the efficient algorithms developed for inverting the stiffness matrix.

Another matrix multiplication of the nodal displacements renders the strains, $\varepsilon_{ij(F)}$, which become constant within the finite element for the important choice of linear shape functions for the distribution of the displacements within the finite element. Nodal displacements are the generalized coordinates in such a Ritz approximation of deformation. According to the definition of impotent eigenstrains, section 4.1.2, these load strains are to be selected for shape control. Since hundreds or thousands of degrees-of-freedom are necessary to reflect the properties of the continuous structure, it becomes necessary in the course of shape control to form hyper elements of proper size and to approximate the strain distribution even further. Subsequently, the procedure is illustrated for idealized trusses, considering both, impotent strains for shape control without additional stress and nilpotent strains for control (or redistribution) of load stress. The general solution of impotent eigenstrains is presented for an in-plane loaded isotropic and homogeneous linear elastic plate when discretized by the FEM. Such a two-dimensional plane stress problem is considered for the sake of simplicity in notation.

4.3.1 Truss Field with Single Redundancy, Force Load Specified

In Fig. 12, the X-braced square shaped idealized truss is shown, specifically loaded by four self-equilibrating forces. To include an anisotropic effect, the extensional stiffness of one diagonal, member rod number 6, is chosen different from the common stiffness of the remaining five rods,

$$C_6 = \alpha\, C, \qquad C = (EA)^{-1} \tag{4.20}$$

Since the analysis of the load case is quite simple, say by application of Menabrea's theorem, see, e.g., Irschik and Ziegler (2001), we list the result on the load strains numbered according to the member rods in Fig. 12 as follows,

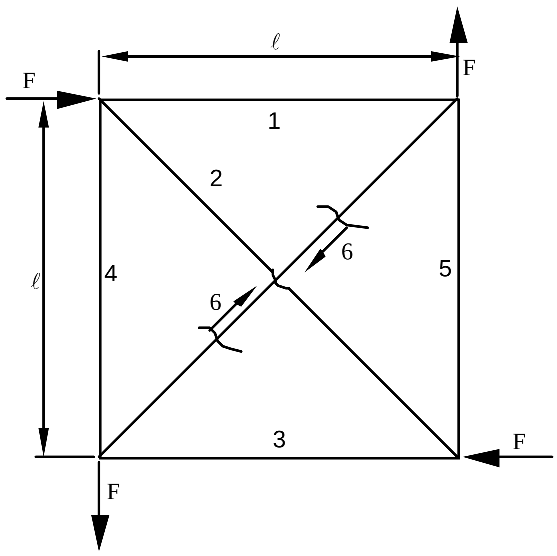

Figure 12: Single redundant truss field under specified self-equilibrating external load F

$$\underline{\varepsilon}_{(F)}^{T} = C\,F\left[\left(-\beta/\sqrt{2}\right),\left(\beta-\sqrt{2}\right),\left(-\beta/\sqrt{2}\right),\left(1-\beta/\sqrt{2}\right),\left(1-\beta/\sqrt{2}\right),\alpha\,\beta\right] \tag{4.21}$$

where

$$\beta = \left(1 + \frac{\alpha}{1+\sqrt{2}}\right)^{-1} \tag{4.22}$$

denotes the anisotropy factor.

It is easily checked that the eigenstrain

$$\bar{\varepsilon}_n = \varepsilon_{n(F)}, \quad n = 1, 2, ..6 \tag{4.23}$$

do not produce any stresses in the member rods, $S_{n(\varepsilon)} = 0,\ n = 1, 2, ..6$. Hence, they are impotent strains and with their sign reverted, $\bar{\varepsilon}_n^{*} = -\varepsilon_{n(F)}$, annihilate the load deformations in the special case illustrated in Fig. 12.

4.3.2 Shape Control of a Truss Field with Single Redundancy and the Nilpotent Strains

The general solution of the impotent eigenstrain is derived by means of inverting the stiffness matrix. The latter is composed from the stiffness of a single member rod of length, ℓ, in local coordinates, say for a plane truss,

$$k = \frac{EA}{\ell}, \ \ \underline{k}_0 = \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix} \tag{4.24}$$

considering the rotational matrix, $\underline{A}$, and the nodal shift matrix, $\underline{B}$, in Hooke's law, $F = ku$,

$$\underline{k}_{ij} = \begin{pmatrix} \underline{k}_{ii} & \underline{k}_{ij} \\ \underline{k}_{ji} & \underline{k}_{jj} \end{pmatrix} = \frac{E_{ij}\,A_{ij}}{\ell_{ij}^3} \begin{pmatrix} x_{ji}^2 & x_{ji}\,y_{ji} & -x_{ji}^2 & -x_{ji}\,y_{ji} \\ \cdot & y_{ji}^2 & -x_{ji}\,y_{ji} & -y_{ji}^2 \\ \cdot & \cdot & x_{ji}^2 & x_{ji}\,y_{ji} \\ \cdot & \cdot & \cdot & y_{ji}^2 \end{pmatrix} \tag{4.25}$$

where,

$$\underline{A} = \frac{1}{\ell_{ij}} \begin{pmatrix} x_{ji} & y_{ji} \\ -y_{ji} & x_{ji} \end{pmatrix}, \underline{B} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \underline{k}_{ii} = \underline{A}^T \underline{k}_0\,\underline{A}, \underline{k}_{ij} = \underline{B}\,\underline{k}_{ii}, \underline{k}_{jj} = \underline{B}\,\underline{k}_{ij} = \underline{k}_{ii} \tag{4.26}$$

have been applied consecutively with the global nodal coordinates taken into account in the differences,

$$x_{ji} = x_j - x_i, \ \ y_{ji} = y_j - y_i \tag{4.27}$$

Application of the general stiffness matrix of a single member (4.25) to the (quadratic) field of Fig. 13 yields the symmetric stiffness matrix, vector of nodal displacements is indicated and the kinematic boundary conditions are built in,

$$\underline{u}^{(1)} = \left(v_b^{(1)} \quad u_c^{(1)} \quad v_c^{(1)} \quad u_d^{(1)} \quad v_d^{(1)}\right)^T$$

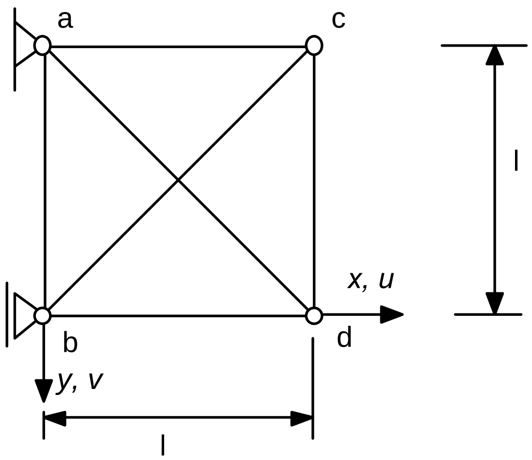

Figure 13: Single redundant truss field, variable stiffness of all six rods, statically determinate support and global coordinates illustrated.

$$\underline{K}^{(1)} = \begin{pmatrix} k_{bc}^{(1)} + k_{ba}^{(1)} & k_{bc}^{(1)} & -k_{bc}^{(1)} & 0 & 0 \\ \cdot & k_{ac}^{(1)} + k_{bc}^{(1)} & -k_{bc}^{(1)} & 0 & 0 \\ \cdot & \cdot & k_{bc}^{(1)} + k_{dc}^{(1)} & 0 & -k_{dc}^{(1)} \\ \cdot & \cdot & \cdot & k_{bd}^{(1)} + k_{ad}^{(1)} & k_{ad}^{(1)} \\ \cdot & \cdot & \cdot & \cdot & k_{ad}^{(1)} + k_{cd}^{(1)} \end{pmatrix} \tag{4.28}$$

where $k_{ij}^{1} = \dfrac{E_{ij}^{(1)} A_{ij}^{(1)}}{\ell_{ij}^{(1)*}}$, $\ell_{ad}^{(1)*} = 2 \cdot \ell_{ad}^{(1)} = \ell\, 2\sqrt{2} = \ell_{bc}^{(1)*}$, the lengths of the outer member rods remain unchanged, $\ell_{ij}^{(1)*} = \ell$.

Inversion of Eq. (4.28) yields the nodal displacements for any load case $\underline{F}$. Choosing consecutively all possible unit nodal force loads yields the matrix of associated nodal displacements, identical to the columns of the flexibility matrix,

$$\underline{u}_{(F)} = \underline{K}^{-1}\underline{F} = \underline{K}^{-1} \tag{4.29}$$

Using so called code-vectors $\underline{n}_{ij}$,

$$\begin{aligned} &\underline{n}_{ab} = (4 \quad 0 \quad 0 \quad 0 \quad 0),\ \underline{n}_{ac} = (0 \quad 3 \quad 4 \quad 0 \quad 0),\ \underline{n}_{ad} = (0 \quad 0 \quad 0 \quad 3 \quad 4), \\ &\underline{n}_{bc} = (2 \quad 3 \quad 4 \quad 0 \quad 0),\ \underline{n}_{bd} = (2 \quad 0 \quad 0 \quad 3 \quad 4),\ \underline{n}_{cd} = (0 \quad 1 \quad 2 \quad 3 \quad 4), \end{aligned} \tag{4.30}$$

the 4x5 matrix of the nodal displacements in global coordinates of all these unit load cases is formed, the indicator of load case (F) is suppressed, thus, for a member rod (ij),

$$\underline{u}_{ij}^{\ T} = (\underline{u}_i, \underline{v}_i, \underline{u}_j, \underline{v}_j) \tag{4.31}$$

that is subsequently transformed to local coordinates by matrix multiplication,

$$\underline{u}_{0ij} = \underline{\hat{A}}_{ij}\, \underline{u}_{ij}, \quad \underline{\hat{A}}_{ij} = \frac{1}{\ell_{ij}} (x_{ij} \quad y_{ij}) \tag{4.32}$$

Consequently, the strain vector of a single member rod between nodes numbered i and j, for all unit load cases, is given by taking the difference and referring it to the rod length,

$$\underline{\varepsilon}_{ij(F)} = \frac{1}{\ell_{ij}}\left(\underline{u}_{0j} - \underline{u}_{0i}\right) \tag{4.33}$$

and the strain matrix of all member rods and all unit load cases

$$\underline{\varepsilon}_{(F)}^{\;T} = \left(\underline{\varepsilon}_{ab} \quad \underline{\varepsilon}_{ad} \quad . \quad . \quad . \quad .\right) \tag{4.34}$$

defines the general solution of impotent eigenstrains $\underline{\bar{\varepsilon}}^{*}$, which are eligible for shape control without additional stress.

Assuming for the sake of simplicity a constant extensional stiffness of the member rods in Fig. 13 and considering imposed eigenstrain in the absence of external force load, the redundant force is selected to be $S_{bc(\varepsilon)} = X_{1(\varepsilon)} = S_{ad(\varepsilon)}$, the nodal equilibrium conditions render the axial forces in the member rods to be proportional, note the statically determinate support conditions,

$$\underline{S}_{(\varepsilon)}^{T} = X_{1(\varepsilon)}\left(-\frac{1}{\sqrt{2}} \quad 1 \quad -\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}} \quad 1 \quad -\frac{1}{\sqrt{2}}\right) \tag{4.35}$$

Hence, there is no size effect, and the nilpotent eigenstrains, causing no deformations (total strains vanish), are distributed according to

$$\underline{\bar{\varepsilon}}^{**} = d_1\left(\frac{1}{\sqrt{2}} \quad -1 \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad -1 \quad \frac{1}{\sqrt{2}}\right)^{T}, \quad d_1 \; \frac{X_{1(\varepsilon)}}{EA} \tag{4.36}$$

Nyashin and Lokhov (2003), defined the inner product of strain tensors in the Hilbert space,

$$\left(\underline{B}.\underline{D}\right) = \int_V \underline{B}.\underline{C}^{-1}\underline{D}\,\mathrm{dV} = \sum_{j=1}^{N} B_j D_j E_j A_j L_j = EA\sum_{j=1}^{N} B_j D_j L_j \tag{4.37}$$

which is also shown reduced to a sum for discrete structures, and further simplified for constant extensional stiffness. Thus, considering the norm of Eq. (4.36), $N = 6$ in Eq. (4.37), the unit base element of the subspace H_σ in Hilbert space of nilpotent strains is defined, its order equals the grade of redundancy,

$$\underline{\phi}_{(1)}^{**} = \frac{1}{\left\|\underline{\bar{\varepsilon}}^{**}\right\|}\underline{\bar{\varepsilon}}^{**} = \frac{1}{\sqrt{EA\ell}}\frac{1}{\sqrt{2+2\sqrt{2}}}\left(\frac{1}{\sqrt{2}} \quad -1 \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad -1 \quad \frac{1}{\sqrt{2}}\right) \tag{4.38}$$

Any given distribution of eigenstrain, when projected on the base unit

$$a = \left(\underline{\bar{\varepsilon}}.\underline{\phi}_{(1)}^{**}\right) = EA\sum_{j=1}^{N} \bar{\varepsilon}_j \phi_{(1)j}^{**} \ell_j \tag{4.39}$$

yields elements in H_σ. Hence, taking the difference renders the impotent part of the eigenstrain distribution, the decomposition into impotent and nilpotent components

$$\underline{\bar{\varepsilon}}^{*} = \underline{\bar{\varepsilon}} - a\underline{\phi}_{(1)}^{**} \tag{4.40}$$

The impotent eigenstrains, $\bar{\varepsilon}^{*}$, form the multidimensional subspace H_u in Hilbert space which is orthogonal to H_σ. Its dimension is determined by the number of unit force loads, hence, in this case, the order is 5, see Eq. (4.31), and see again Fig. 13.

Limitations of load stress redistribution by nilpotent eigenstrains is clearly indicated by Eq. (4.36) where size and sign of the parameter d_1 can be selected with the distribution of strain and thus stress, Eq. (4.35) given.

4.3.2.1 Decomposition of Imposed Eigenstrains

Say, the diagonal member rod in Fig. 13 only is subjected to an imposed eigenstrain, $\bar{\varepsilon}_{bc}$. From Eq. (4.39)

$$a_1 = -\sqrt{EA\ell}\,\frac{\bar{\varepsilon}_{bc}}{\sqrt{1+\sqrt{2}}} \tag{4.41}$$

results in this case, and the nilpotent part of the eigenstrain vector $\bar{\varepsilon}$ becomes,

$$\bar{\varepsilon}^{**} = a_1 \phi^{**}_{(1)} = \frac{\bar{\varepsilon}_{bc}}{2+\sqrt{2}}\begin{pmatrix} -\frac{1}{\sqrt{2}} & 1 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 1 & -\frac{1}{\sqrt{2}} \end{pmatrix}^T \tag{4.42}$$

Equation (4.40) determines the remaining part of the impotent eigenstrain vector.

4.4 The Dynamic Problems of Shape Control

The main goal of dynamic shape control is vibration suppression by means of controlled input of electrical voltage into shaped piezoelectric layers. The latter function as both, sensors and actuators. These piezoelectric strains are the (in a relaxed sense, impotent) eigenstrains imposed in a time specific manner to annihilate the total transient strains, i.e., within the state-of-the-art, without production of additional quasi-static stress. However, also a counterpart of the static nilpotent eigenstrains exist and a control of the quasistatic force stresses seems to be possible with negligible perturbation of the dynamic shape control. Force induced, linear elastic vibrations about the equilibrium state are annihilated by proper transient thermoelastic strains by Irschik and Pichler (2001). The same authors improved their analysis considerably in Irschik and Pichler (2003). The generalization, of special importance for applications in aeronautics, to consider small vibrations of a structure superposed on large rigid body motion is performed by Irschik et al.(2003) with further improvements contained in Irschik et al. (2004). These solutions are based on the force load known in advance (open loop control). For the control of plate vibrations in this context, see Irschik et al. (1997d). Rotary wings are considered by Nitzsche and Breitbach (1994). Further aspects of automatic control are summarized by Schlacher and Kugi (1999) and Kugi (2001). Preumont (2002) provides a reach selection. Application of tendons or hydraulic actuators in large scale structures due to their discrete nature are not further discussed in this paper, see Soong (1990). However, for adaptive wings see Austin et al. (1994).

4.4.1 Suppression of Force-induced Small Vibrations about an Equilibrium State

The generalization of static shape control to dynamic shape control can be based on the dynamic generalization of Maysel's formula (4.11) to the dynamic case of eigenstrain loads, see Irschik et al. (1993) where the dynamic Greens stress dyadic is applied and a convolution in time must be considered. When case A refers to eigenstrains which are here denoted ε^{*A}, and the dummy force problem is denoted B, in the Laplace transform domain (the overbar denotes the Laplace transformed quantity and convolution is understood) and for quiet initial conditions, the latter are assumed throughout of this section, the dynamic version of Maysel's formula becomes, note the change of notation against Eq. (2.8),

$$\int_V \bar{b}_i^B \bar{u}_i^A \,\mathrm{dV} = \int_V \bar{\sigma}_{kl}^B \bar{\varepsilon}_{kl}^{*A} \,\mathrm{dV} \tag{4.43}$$

Since additional terms appear in Eq. (4.43), special care must be taken in case of nonhomogeneous initial conditions when applying the control.
Irschik and Ziegler (1988), split the response A to the eigenstrain load into the quasistatic one, to be solved by any efficient method and the remaining dynamic one using the static Green's force function in Eq. (4.43).

4.4.1.1 Force-induced Small Vibrations about an Equilibrium State

Irschik and Pichler (2001) and (2003), solve the dynamic shape control problem assuming the forced vibrations known by assigning proper actuator stresses, i.e., transient eigenstresses. They elegantly use an extension of Neumann's method in Irschik and Pichler (2003), to define the properly distributed actuators. The force load must be considered first. Since mass inertia is taken into account, Eq. (4.1) becomes nonhomogeneous, rendering conservation of momentum in the form of the Euler-Cauchy equation of motion,

$$\operatorname{div}\underline{\sigma} + \underline{b} = \rho \ddot{\underline{u}} \tag{4.44}$$

The boundary condition (4.2a) remains unchanged, the boundary condition (4.2b) prescribes the time dependent traction. The more general results derived in Irschik and Pichler (2001) and (2003), for dynamic control of the deformation $\underline{u}(\underline{x},t) = \underline{u}^{(F)}$ produced by prescribed body forces and surface traction, allows to control the much simpler quasistatic solution of the force problem, determined by the successive equilibrium states,

$$\operatorname{div}\underline{\sigma}_{(s)}^{(F)} + \underline{b} = \underline{0} \tag{4.45}$$

with boundary condition (4.2b) taken into account. To fully relate the solution to the static shape control, the kinematic boundary condition (4.2a) is applied and the simplifying assumption of both, body force and traction to be separable in space and time, is made. This assumption implies that Eq. (4.45) has to be solved only once and/or all available static solutions become candidates for dynamic shape control.

Since it is common practice in structural dynamics and modal analysis, to split the response to the force loading into its quasistatic part, already posed by the boundary value problem of Eq. (4.45), the complementary dynamic part in the solution, $\underline{u}_{(d)}^{(F)}$,

$$\underline{u}^{(F)} = \underline{u}_{(s)}^{(F)} + \underline{u}_{(d)}^{(F)} \tag{4.46}$$

is considered further. Subtracting Eq. (4.45) from Eq. (4.44) renders the latter in reduced form, note the "body force" $\underline{b}_{(s)}^{*}$, determined by substituting Eq. (4.46) and, consequently recognized, as the inertia force of the quasistatic force solution, with time-invariant spatial distribution,

$$\operatorname{div}\underline{\sigma}_{(d)}^{(F)} + \underline{b}_{(s)}^{*} = \rho \ddot{\underline{u}}_{(d)}^{(F)}, \quad \underline{b}_{(s)}^{*} = -\rho \ddot{\underline{u}}_{(s)}^{(F)} \tag{4.47}$$

Homogeneous dynamic boundary conditions apply in the complementary dynamic problem,

$$\Gamma_{\sigma}: \underline{\sigma}_{(d)}^{(F)} . \underline{n} = \underline{0} \tag{4.48}$$

The kinematic boundary condition (4.2a) still holds true.

Since Eq. (4.45) describes successive states of equilibrium, the definition of the transient eigenstrain distribution by

$$\underline{\varepsilon}_{(s)}^{(F)} + \bar{\underline{\varepsilon}} = \underline{0} \tag{4.49}$$

renders, analogous to the static problem setting, at first annihilation of the quasistatic force deformations according to Eqs. (4.5) and (4.11). However, the quasistatic part of the eigenstrain produced stresses vanish since Eq. (4.12) holds true. That is a relaxed condition for the eigenstrain to be impotent. Inertia of mass renders dynamic stresses produced by the transient eigenstrains.

4.4.1.2 Eigenstrain-induced Small Vibrations and Dynamic Shape Control

Since $\underline{\sigma}_{(s)}^{(\varepsilon)} = \underline{0}$, the trivial solution of the homogeneous Eq. (4.6) for the quasistatic impotent eigenstrain, Eq. (4.49) is understood, the dynamic boundary value problem of the eigenstrain load is determined by

$$\operatorname{div}\underline{\sigma}_{(d)}^{(\varepsilon)} + \underline{b}_{(s)}^{*} = \rho\,\ddot{\underline{u}}_{(d)}^{(\varepsilon)}, \quad \underline{u}^{(\varepsilon)} = \underline{u}_{(s)}^{(\varepsilon)} + \underline{u}_{(d)}^{(\varepsilon)}, \quad \underline{u}_{(s)}^{(\varepsilon)} = \underline{u}_{(s)}^{(F)} \tag{4.50}$$

with homogeneous dynamic boundary conditions,

$$\Gamma_{\sigma}{:}\,\underline{\sigma}_{(d)}^{(\varepsilon)}.\underline{n} = \underline{0} \tag{4.51}$$

and the homogeneous kinematic boundary condition of (4.2a) assigned. Considering at this moment the positive Eq. (4.49) as already inserted in Eq. (4.50), reproduces one and the same body force distribution, defined in Eq. (4.47), and thus renders the solution of the dynamic part of the force problem, Eqs. (4.47) and (4.48). Hence, it can be concluded that the quasistatic impotent eigenstrain, defined in Eq. (4.49), annihilates also the dynamic part of the force-displacements and, in addition, counteracts the dynamic stress portion of the force problem. The ideal dynamic shape control, thus suppresses force-induced vibrations (quiet initial conditions have been assumed throughout) and leaves the quasistatic force-produced stresses unchanged. A simple example illustrates this solution technique based on the quasistatic force response. Redistribution of the remaining quasistatic stress state by imposing quasistatic nilpotent eigenstrains is not within the scope of this paper.

4.4.1.3 Illustrative Example: Suppression of Forced Vibrations

A homogeneous and isotropic linear elastic thick square steel-plate (in plane strain) with two adjacent boundaries rigidly built-in is considered under the action of a uniform load, either instantly switched on, or varying time-harmonically, see Fig. 14. The separated time function is the Heaviside step function in the transient problem or the sinusoidal function in the stationary case, thus, the quasistatic problem, Eq. (4.45) with body force absent, has to be solved only once. The FEM code ABAQUS, standard V6.3, with 3600 plain strain elements CPE4R, provided the averaged principal force-strains, results are plotted in selected points in Fig. 14 for a static unit load. The dynamic problem is solved by applying ABAQUS for the force loads and for the quasistatic impotent eigenstrain load defined by Eq. (4.49), respectively. The latter is interpreted by thermal strains with a proper (fictitious) anisotropic thermal expansion tensor assigned. The latter is assigned to each element of the FE-model using a Visual C++code for pre-processing. Quiet initial conditions apply for the transient problem. The displacement responses in the interior point P_5 for the unit step load are shown in Figs. 15 and 16, those at the edge point P_2 are illustrated in Figs. 17 and 18. The transfer functions of these displacements are shown in Figs. 19 and 20 in the interior point and in Figs. 21 and 22 at the edge point. $T_1 = f_1^{-1}$ is the natural fundamental period of the steel plate. Excellent coincidence of the simulations is observed and, with reversed sign, the forced vibrations would be annihilated. However, note the large number of elements necessary for the eigenstrain load. All the numerical results are supplied by courtesy of Dr. Uwe Pichler, Center of Mechatronics, University of Linz, Austria.

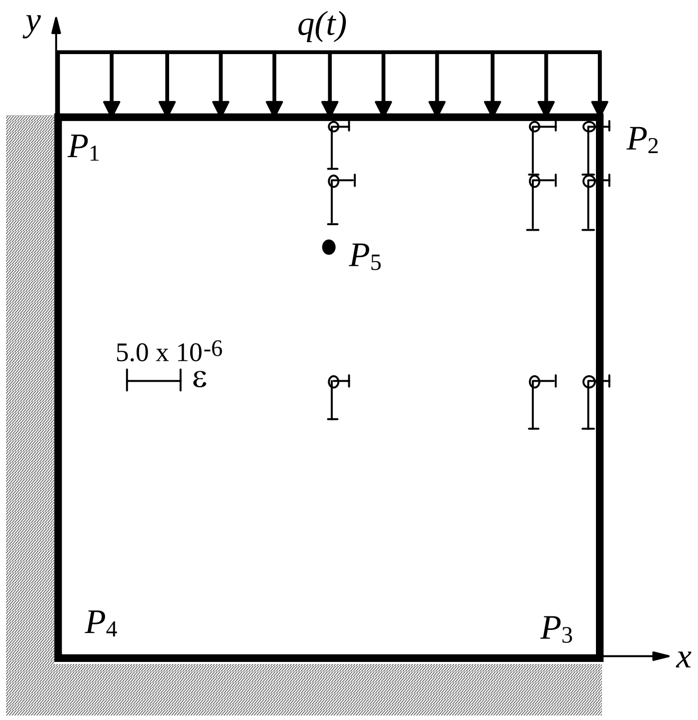

Figure 14: Thick square plate of unit length, under uniform load, in plane strain. Static results for unit force load of scaled principal strains and quasistatic impotent eigenstrains in nine points. Coordinates of Point P_5, $(0.5, 0.75)$

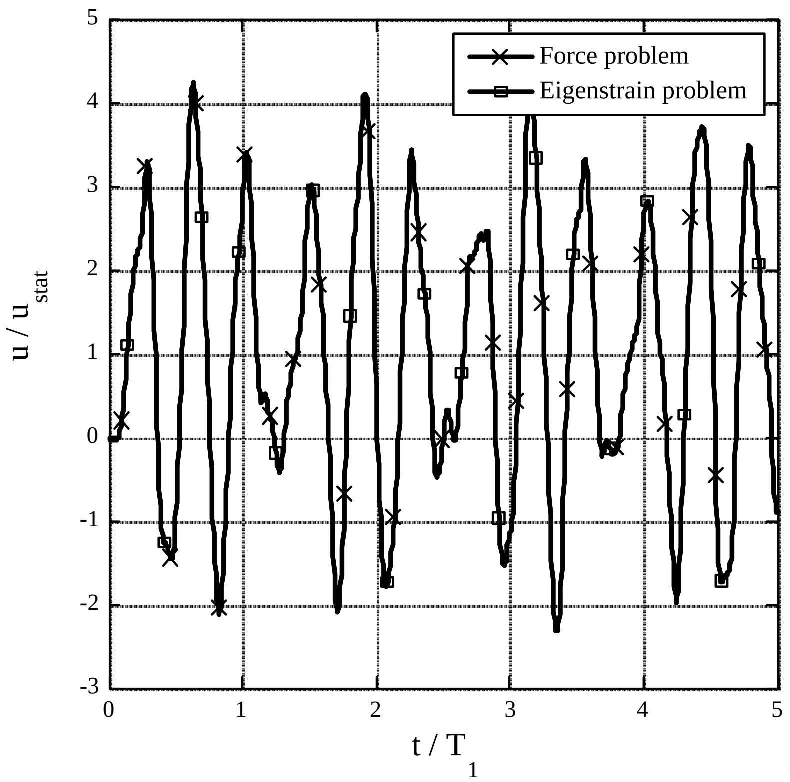

Figure 15: Nondimensional transient horizontal displacement at point P_5.

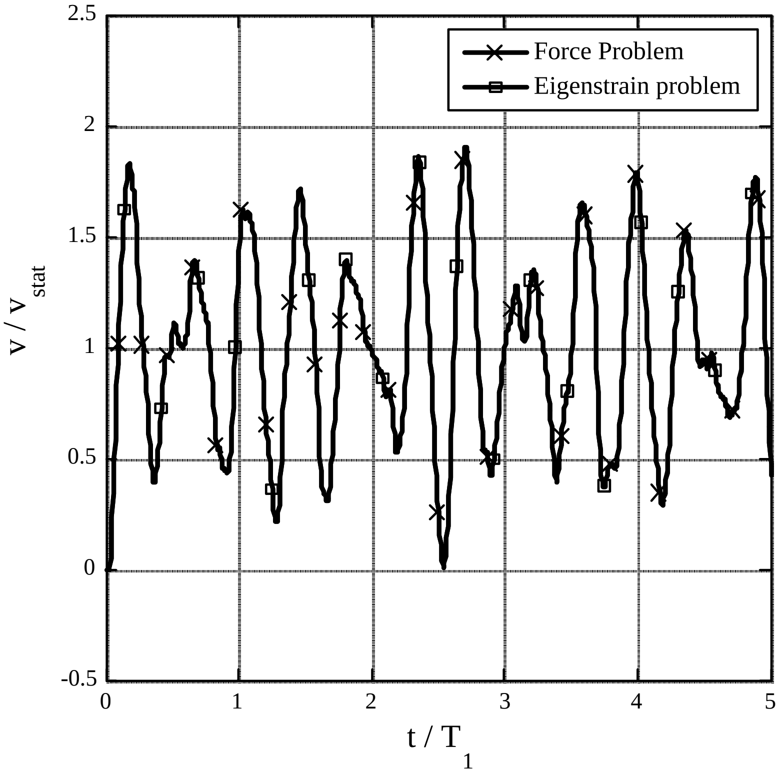

Figure 16: Nondimensional transient vertical displacement at point P_5.

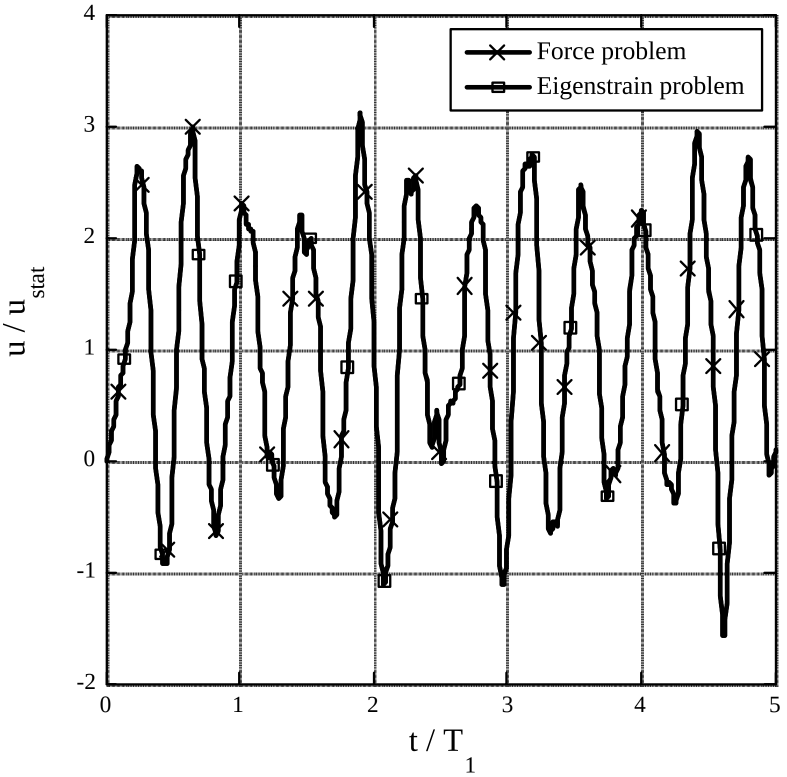

Figure 17: Nondimensional transient horizontal displacement at the edge point P_2.

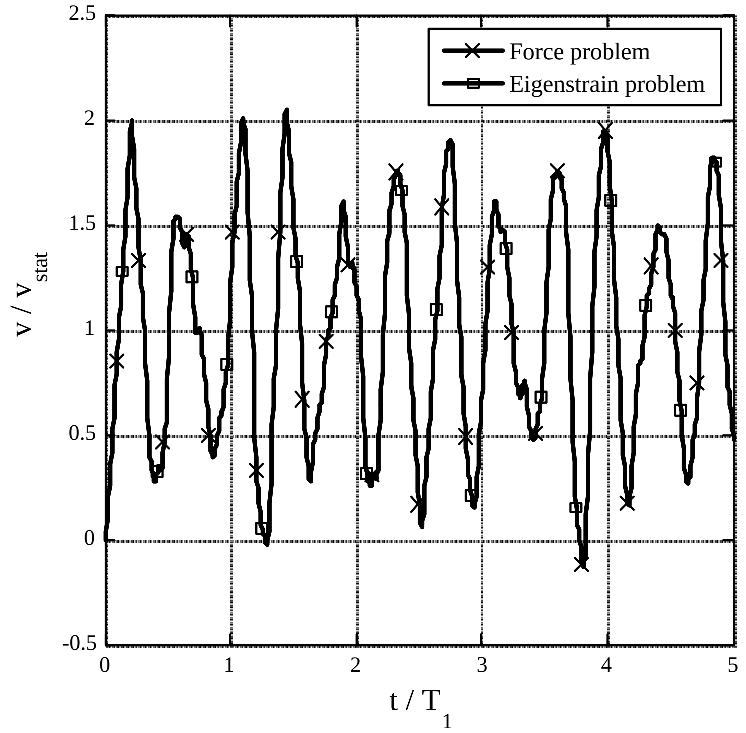

Figure 18: Nondimensional transient vertical displacement at the edge point P_2.

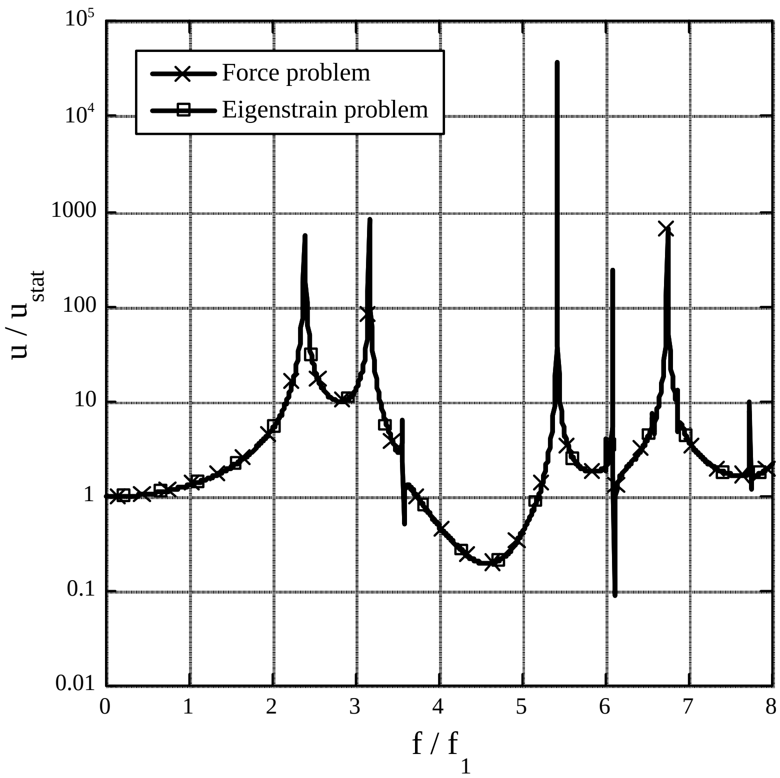

Figure 19: Nondimensional transfer function of the horizontal displacement in point P_5 .

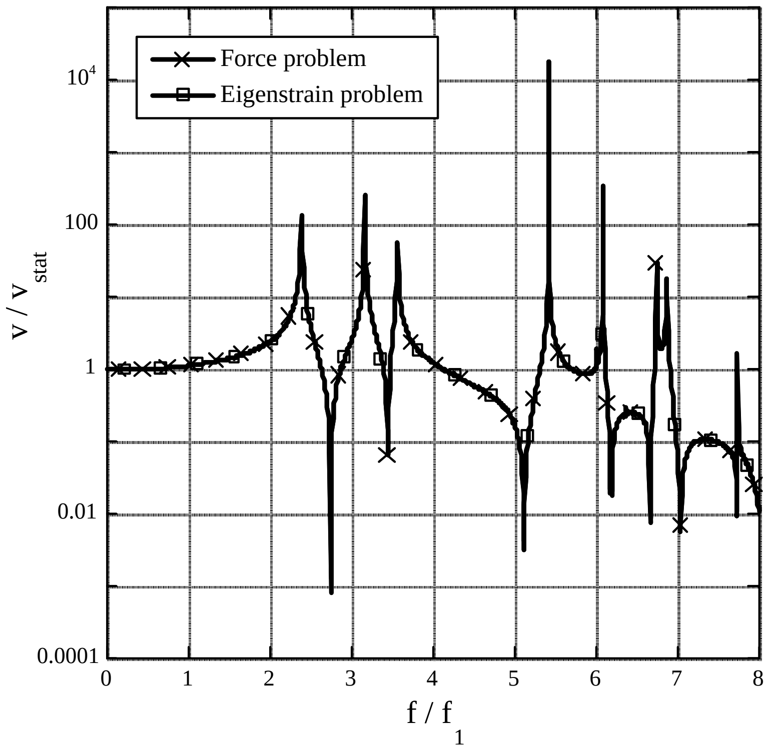

Figure 20: Nondimensional transfer function of the vertical displacement in point P_5.

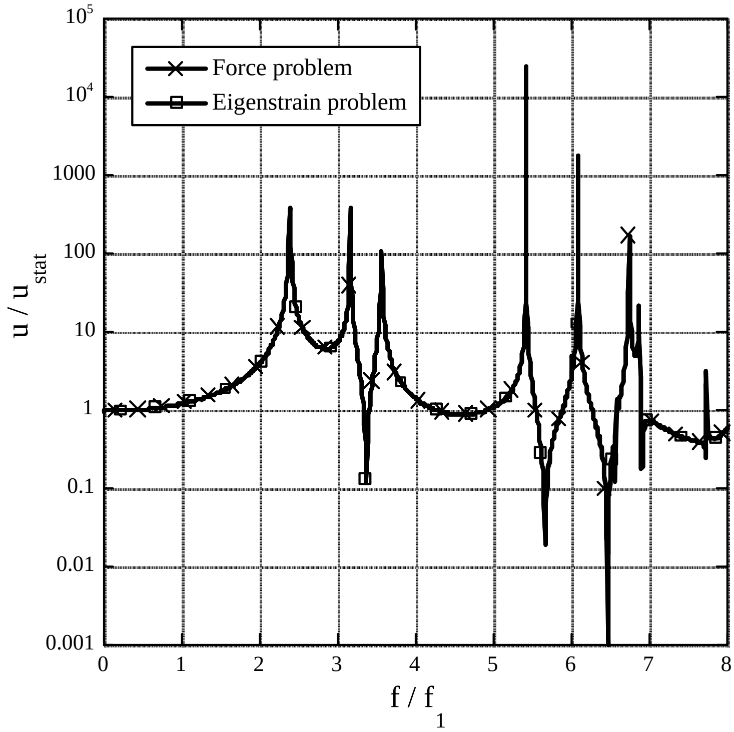

Figure 21: Nondimensional transfer function of the horizontal displacement in the edge point P_2.

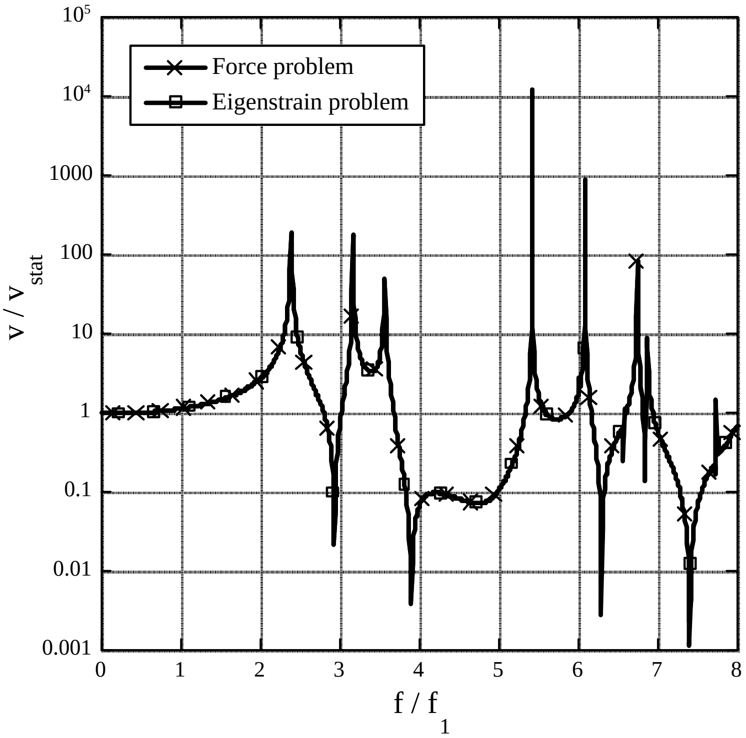

Figure 22: Nondimensional transfer function of the vertical displacement in the edge point P_2.

5 Elastic-Plastic Waves. Application of the Dynamic Maysel's Formula

In this section a short account of the multiple field concept in physical nonlinear dynamics is given when the dynamic Green's function of the background is "readily" available. Equation (2.8) becomes an effective tool to observe the waves emitted from strain determined sources since all properties of the wave guide are contained in the Green's function of the background. For localized sources even the volume integral can be saved by just multiplying the integrand by the small volume of the cell.

5.1 Uniaxial Waves

Elastic-viscoplastic uniaxial waves in thin rods are considered by means of the elastic background with additional plastic sources acting in the transient, progressing plastic zone in Irschik and Ziegler (1990) and, with adiabatically deposited heat in Irschik and Ziegler (1995). Irschik et al (1997c) considered a viscoelastic material. In the multiple field concept a linear background is considered and in the plastic zone, every localized plastic source emits elastic waves, in both directions of the rod, if an increment of plastic strain (kept constant within a single cell of sufficiently small length) is caused within the time step. Causal superposition of the elastic wave pattern produced in the elastic background rod by the external load (e.g., an impact at one end) and of the elastic waves emitted from the plastic sources produces the fully nonlinear solution eventually including plastic shock fronts and eventually the unloading wave. The basic equations are in consecutive order, generalized Hooke's law, and the resulting nonhomogeneous wave equation of the axial displacement u

$$\dot{\sigma} = E\left(\dot{\varepsilon} - \dot{\bar{\varepsilon}}\right), \qquad u_{,xx} - c^{-2}u_{,tt} = \bar{\varepsilon}_{,x}, \qquad c^2 = E/\rho \tag{5.1}$$

Boundary conditions and the material law of the rod are understood to be given in the course of the full solution.

The waves emitted from a concentrated unit plastic source,

$$\bar{\varepsilon}_{,x} = \delta_{,x}\left(x - \xi\right)\delta\left(t - \tau\right) \tag{5.2}$$

are, however, given by the stress influence function of the infinite rod. The force Green's function is a D'Alembert box-type wave of amplitude $c/2E$, hence, the stress takes on the form of two Dirac pulses propagating in opposite directions,

$$\tilde{\sigma}(x,\xi;t,\tau)=\frac{c}{2}\{\delta[x-\xi-c(t-\tau)]-\delta[x-\xi+c(t-\tau)]\}H(t-\tau) \tag{5.3}$$

Distributions of plastic sources render, the particular solution of the nonhomogeneous Eq. (5.1) takes on the integral form (of the dynamic and generalized Maysel's formula for uniaxial stress, Eq. (2.8) properly specified), homogeneous boundary conditions (b.c.) apply, - for convenience, the unit and instantaneous force is now applied at x , -note the displacement Green's function of a unit plastic source of the infinite rod, $\tilde{\sigma}(\xi,x;t)=\tilde{u}(x,\xi;t)$, and thus the complementary form of Maysel's integral is again apparent,

$$u^*(x,t)=\int_0^t \mathrm{d}\tau\int_0^\ell \tilde{\sigma}(\xi,x;t,\tau)\bar{\varepsilon}(\xi,\tau)A\,\mathrm{d}\xi \tag{5.4}$$

The nonhomogeneous boundary conditions render the D'Alembert wave $u_0(x,t)$ that is a solution of the homogeneous wave equation in the elastic background, and thus not discussed further. The total displacement becomes incrementally,

$$\Delta u=\Delta u_0(x,t)+\Delta u^*(x,t) \tag{5.5}$$

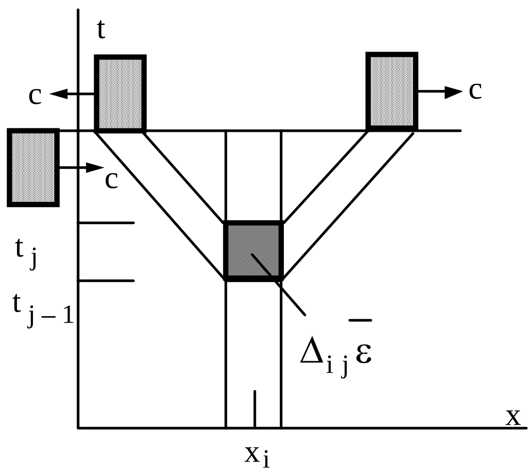

Figure 23: Acoustic emission from a local plastic source in a semi-infinite rod, stress wave and reflection shown.

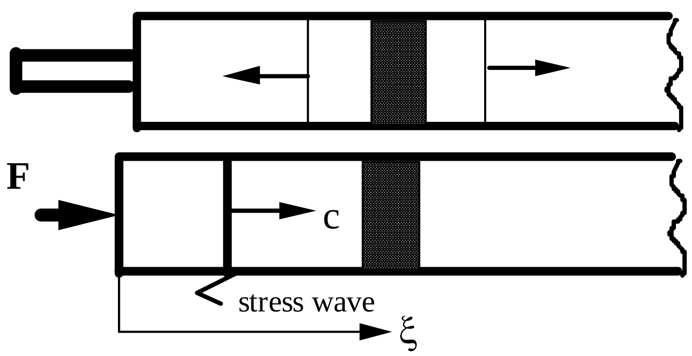

Figure 24: Transducer in receiving mode attached to a rod with a local plastic event. Stress wave of Green's function of the second kind, carrying all information on the background wave guide, illustrated, for convolution with the plastic strain increment

Figure 23 shows the Mach plane of the action of a single plastic source with a stress wave reflection at one finite boundary, say $x = 0$, indicated, i.e. the stress Green's function of the infinite rod was used in Eq. (5.4) in that simple case. Green's function of the semi-infinite rod could be substituted to account for the reflected wave as well. If the plastic zone is initiated at the impacted boundary, the limiting process $x \to 0$ must be performed to render the Green's function of the second kind (unit force applied at the boundary). For general time function of the force, when applied at x, or $x = 0$, respectively, we note the Laplace transformed Green's stress waves accordingly,

$$\varepsilon = \pm 1 : \tilde{\sigma}(\xi, x, s) = -\frac{\varepsilon}{2}\frac{F(s)}{A}\exp -\frac{s}{c}|\xi - x|, \qquad \tilde{\sigma}(\xi, s) = -\frac{F(s)}{A}\exp -\frac{s}{c}\xi \tag{5.6}$$

The latter Green's function of the second kind must be substituted in Eq. (5.4) if the wave emitted from an isolated plastic source is to be observed say in a transducer in receiving mode as shown in Fig. 24, a situation designed for monitoring ductile structures which might be subjected to overloads, see Ziegler (2001) for details.

5.2 Waves Emitted from a Localized Plastic Source in a Layered Plate or Half space

Equation (2.8) is extremely suitable to describe the waves emitted from a localized strain determined source if the dynamic Green's function of the background waveguide is readily available. For a layered half space or plate, the three dimensional Green's dyadic has been constructed in terms of generalized ray integrals (i.e. an expansion into plane waves) in Pao and Gajewski (1977). For the sake of monitoring such ductile composite structures with transducers placed on the free surface, the Green's stress dyadic of the second kind must be substituted in Eq. (2.8). The solution in the Laplace transform domain has been worked out in Ziegler and Borejko (2000) and detailed by Ziegler (2001), with nonhomogeneous initial conditions considered, note the change of notation with respect to Eqs. (2.8) and (5.4), point of observation is subsequently $\boldsymbol{x}_0$, commonly placed on the free surface,

$$\begin{aligned} u_i(\boldsymbol{x}_0; s) &= \int_V \tilde{\sigma}_{\alpha\beta(i)}(\boldsymbol{x}, \boldsymbol{x}_0; s)\bar{\varepsilon}_{\alpha\beta}(\boldsymbol{x}; s)\mathrm{dV}(\boldsymbol{x}) \\ &+ \rho\int_V \tilde{u}_{\alpha(i)}(\boldsymbol{x}, \boldsymbol{x}_0; s)\left[s\bar{u}_\alpha(\boldsymbol{x}; t=0) + \dot{\bar{u}}_\alpha(\boldsymbol{x}; t=0)\right]\mathrm{dV}(\boldsymbol{x}) \end{aligned} \tag{5.7}$$

Working with the plastic sources distributed in the transient plastic zone considerably improved the attempts of Jimma and Masuda (1978) to solve the problem by a source concentrated at the wave front. For the numerical analysis of a point symmetric problem see again Fotiu and Ziegler (1996).

The displacement potentials of the Helmholtz decomposition of the Green's displacements of the second kind, a concentrated force is applied orthogonal to the free surface, say $\boldsymbol{x}_0 = \boldsymbol{0}$, which is the location of the transducer in receiving mode,

$$\bar{\phi}(x, y, z, s) = s^2\,\bar{F}(s)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_P \exp(s\,g_P)\mathrm{d}\xi\,\mathrm{d}\kappa \tag{5.8}$$

$$\bar{\psi}_x = s^2\,\bar{F}(s)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Sx} \exp(s\,g_S)\mathrm{d}\xi\,\mathrm{d}\kappa \tag{5.9}$$

$$\bar{\psi}_y = s^2\,\bar{F}(s)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Sy} \exp(s\,g_S)\mathrm{d}\xi\,\mathrm{d}\kappa \qquad \bar{\psi}_z(x, y, z, s) = 0 \tag{5.10}$$

with emittance functions and phases given in terms of the wave slowness of the apparent plane waves and their (potential) reflection coefficients at the free surface, see Borejko (1996) for the latter,

$$S_P = \left[\left(-1 + R^{PP}\right) - R^{S_xP}\frac{i\kappa}{\varsigma} + R^{S_yP}\frac{i\xi}{\varsigma}\right], \qquad -t = g_P = i\xi x + i\kappa y - \eta z$$

$$S_{Sx} = \left[-\left(1 + R^{S_xS_x}\right)\frac{i\kappa}{\varsigma} + R^{PS_x} + R^{S_yS_x}\frac{i\xi}{\varsigma}\right], \quad S_{Sy} = \left[\left(1 + R^{S_yS_y}\right)\frac{i\xi}{\varsigma} + R^{PS_y} - R^{S_xS_y}\frac{i\kappa}{\varsigma}\right]$$

$$-t = g_s = i\xi x + i\kappa y - \varsigma z$$

represent the source ray (direct ray) in a point $z < h$ of a homogeneous surface layer of thickness h. The body wave speeds are denoted $c > C$.

A "receiver" at a point $(x, y, z = h)$ in the interface to a dissimilar layer or half space, requires at first the definition of the potentials of the reflected rays for the point $z < h$, to be superposed to those of the source ray. Subsequently displacements and stresses are determined and the limiting process $z \to h$ renders finally so called receiver functions. The resulting Helmholtz potentials are,

$$\overline{\varphi}(x, y, z \le h, s) = \overline{\phi} + \overline{\varphi_{pP}} + \overline{\varphi_{s,xP}} + \overline{\varphi_{s,yP}} \tag{5.11}$$

$$\overline{\varphi_{pP}} = s^2\, \overline{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_P\, R_{PP} \exp\left(s\, g_{pP}\right) \mathrm{d}\xi\, \mathrm{d}\kappa, \quad g_{pP} = i\,\xi\, x + i\,\kappa\, y - \eta\, h - \eta\,(h - z) \tag{5.12}$$

$$\overline{\varphi_{s,kP}} = s^2\, \overline{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{S,k}\, R_{S,kP} \exp\left(s\, g_{sP}\right) \mathrm{d}\xi\, \mathrm{d}\kappa, \quad k = x, y \tag{5.13}$$

$$g_{sP} = i\,\xi\, x + i\,\kappa\, y - \varsigma\, h - \eta\,(h - z), \quad g_{pS} = i\,\xi\, x + i\,\kappa\, y - \eta\, h - \varsigma\,(h - z)$$

$$\overline{\psi_{S,j}}(x, y, z \le h, s) = \overline{\psi_j} + \overline{\psi_{pS,j}} + \overline{\psi_{s,xS,j}} + \overline{\varphi_{s,yS,j}}, \quad j = x, y, z \tag{5.14}$$

$$\overline{\psi_{pS,j}} = s^2\, \overline{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_P\, R_{PS,j} \exp\left(s\, g_{pS}\right) \mathrm{d}\xi\, \mathrm{d}\kappa, \quad j = x, y, \quad R_{S,zP} = R_{PS,z} = 0 \tag{5.15}$$

$$\overline{\psi_{s,kS,j}} = s^2\, \overline{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{S,k}\, R_{S,kS,j} \exp\left(s\, g_{sS}\right) \mathrm{d}\xi\, \mathrm{d}\kappa, \quad k = x, y, \quad j = x, y, z \tag{5.16}$$

$$g_{sS} = i\,\xi\, x + i\,\kappa\, y - \varsigma\, h - \varsigma\,(h - z), \quad \eta = \sqrt{c^{-2} + \xi^2 + \kappa^2}, \quad \varsigma = \sqrt{C^{-2} + \xi^2 + \kappa^2}$$

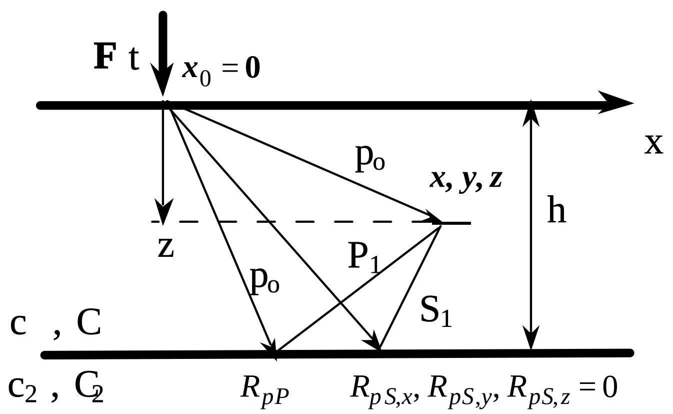

Figure 25: P-wave emitted from a vertical single surface force and received at a buried receiver (where a local plastic source is expected). A plastic interlayer slip requires the limit of receiver depth taken to the layer thickness, $z \to h$.

	σ_{xy} / μ		
$\bar{\Sigma}_{xy}^{P} / s^2$	$\left(1 + R_{pP}\right)\left(-2\,\xi\kappa\right) - R_{pS,y}\left(2 i\kappa\zeta\right)$	S_p	g_p
$\bar{\Sigma}_{xy}^{Sx} / s^2$	$R_{s,x\,P}\left(-2\xi\kappa\right) - R_{s,xS,y}\left(2\,i\,\kappa\zeta\right) + R_{s,xS,z}\left(-\zeta^2 + \xi^2 - \kappa^2\right)$	$S_{s,x}$	g_s
$\bar{\Sigma}_{xy}^{Sy} / s^2$	$\left(1 - R_{s,yS,y}\right)\left(2 i\kappa\zeta\right) + R_{s,y\,P}\left(-2\,\xi\kappa\right) + R_{s,y\,S,z}\left(-\zeta^2 + \xi^2 - \kappa^2\right)$	$S_{s,y}$	g_s
$\bar{\Sigma}_{xy}^{Sz} / s^2$	$\left(1 + R_{s,z\,S,z}\right)\left(-\zeta^2 + \xi^2 - \kappa^2\right) - R_{s,z\,S,y}\left(2 i\kappa\zeta\right)$	$S_{s,z}$	g_s

Table 1: Stress receiver functions at interior (plastic source) point, $\mu = \rho\, C^2$.

Figure 25 illustrates the geometric ray paths with one reflection at the interface (defining the arrival times). For the numerical procedure of inverting the Laplace transforms, see Borejko et al. (2001). For convenience, we present the receiver functions of one interface shear stress component in Table 1, a full account is given in Ziegler (2002).

6 Conclusions

The multiple field theory reviewed in this paper is derived from the classical two-field theory of linearized thermo-elasticity with an isothermal background structure. In Ch. 6 of Ziegler (2002), Maysel's formula is derived from the principle of virtual work (virtual displacements) and specialized to keep rotational symmetry intact or to account for simplifications of its application to thin-walled structural members. A third field of eigenstrains imposed on the background material results from structural thermo-viscoplasticity under linearized geometric conditions, recent reports in Applied Mechanics Reviews are Allen (1991), Jones (1989) and Ohno (1990). For certain materials, ductile damage becomes important and is easily incorporated within the analysis. Due to the (material) nonlinearity, an incremental formulation becomes necessary.

Proper extensions of Maysel's formula are discussed, which render an optimal solution strategy with either the static or the dynamic Green's function of an auxiliary problem taken into account. Nonlinear vibration problems in the lower frequency band are solved by expansion in the linear modes of the background structure. However, the quasi-static response is separated and only the remaining dynamic portion is subjected to this approximation thereby rendering a stable numerical algorithm. The spatial distribution of the internal sources and their time evolution are determined from the specific constitutive relation. Since the solution is available in general terms, material parameters may be changed or several material laws may be considered without much additional numerical effort. The plastic drift results as a natural byproduct of splitting the solution. The problems discussed so far have been selected in connection with the important research results and interests of Fred Rimrott, which he achieved mainly in connection with space structural applications.

In general, elastic-viscoplastic waves require the use of the dynamic Green's function. The elastic wave pattern in the background structure is given in terms of convolution integrals. With proper time spline functions for the increments of the eigenstrain, integration is analytically performed, and the waves are qualitatively known. Again, the constitutive relations determine the intensity of the internal sources. An impacted semi-infinite viscoplastic and damaging rod with transient temperatures developing in the plastic zone under adiabatic conditions serves as an illustration of the multiple field theory for that class of physically nonlinear dynamic problems. Without making use of the speed of any plastic shocks, the superposition of the incremental elastic wave pattern includes such nonlinear phenomena. However, sharp wave fronts may be somewhat rounded off. Some sensitivity of the elastic forerunner was observed on the choice of the spatial cell size. However, the tail of the plastic wave is quite insensitive.

The multiple field theory not only provides an optimal solution strategy especially for rather general vibration problems but allows changing of material parameters or playing a material game at low computational costs.

An extension of the multiple field theory to include geometric nonlinearity is possible for some special vibration problems. If such a generalization is possible for plastic waves in simple structures will be seen.

Last but not least, such a multiple field theory supplies ideally the proper fields of eigenstrain in optimal deformation control of linear elastic structures. Distributed actuators are understood. Fred Rimrott's interest in this subject relied mainly on deployable structures.

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