Linear Programming for a Cutting Problem in the Wood Processing Industry – A Case Study

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Abstract: In this paper the authors present a case study from the wood-processing industry. It focuses on a cutting process in which material from stock is cut down in order to provide the items required by the customers in the desired qualities, sizes, and quantities. In particular, two aspects make this cutting process special. Firstly, the cutting process is strongly interdependent with a preceding handling process, which, consequently, cannot be planned independently. Secondly, if the trim loss is of a certain minimum size, it can be returned into stock and used as input to subsequent cutting processes. In order to reduce the cost of the cutting process, a decision support tool has been developed which incorporates a linear programming model as a central feature. The model is described in detail, and experience from the application of the tool is reported.

Keywords: one-dimensional cutting, linear programming, wood-processing industry.

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Table of Contents

1 Introduction

Planning and control of industrial cutting processes are classic application areas of operations research. Many publications were written about this topic, the earliest dating back to the 1950s. However, even though cutting processes in general can be considered as thoroughly researched types of production processes, the number of yearly publications in this field still seems to be growing. This is due to the fact that these processes still hold a high potential for efficiency improvement.

Cutting problems from practice are often characterized by specific goals and constraints which do not allow the application of standard models and solution algorithms. Instead, such methods have to be modified in an appropriate way, or even completely new ones have to be developed in order to cope with real-world requirements. An application of this kind from the wood processing industry will be presented in this paper. The respective cutting problem is special with respect to two aspects. On the one hand, the cutting process is strongly linked to preceding transport and inventory processes, which cannot be neglected when planning the cutting process. On the other hand, trim loss does not necessarily represent waste. Instead, large pieces of trim loss may be put back into stock and used later in order to satisfy customer orders in the future.

The paper will be organized as follows: In the following chapter 2, the company and the planning situation under consideration are explained. Chapter 3 presents the analysis of the problem with regard to the decision-relevant costs and discusses the question whether all cost factors are ascertainable. In chapter 4 the chosen solution approach and the developed optimization model are explained. Chapter 5 gives some information about the implementation of the solution. Finally, in chapter 6 first experiences and results from the usage of the decision support system are reported and future developments are outlined.

2 The Cutting Problem

Nordlam GmbH is an internationally-operating company from the wood processing industry, which produces glued laminated timber (so-called gluelams) in various sizes and qualities at its Magdeburg plant. These products can be regarded as wooden beams, which are manufactured by gluing together wooden boards (lamellas) in a multi-stage production process. They are used in industry buildings like warehouses and production plants, but also – in an increasing number of cases – in private homes.

An incoming customer order will at first be analyzed whether a production order has to be released or whether it is possible to satisfy the order directly from the inventory of finished products, which is available in the warehouse. Generally speaking, large customer orders will result in new production orders, while small ones are satisfied directly from the existing inventory which will be cut down into the requested sizes. This second case will be considered in this paper.

Planning of the cutting process is always related to product families, i.e. products which can be grouped together with respect to characteristics such as width, height, and quality. At first, for each specific customer order (or a set of customer orders which belong to the same product family) all the finished products available in the warehouse from which the order can be satisfied have to be determined. There are two types of finished products available, namely standard material, which comes in lengths of 24 meters and originates – as excess production – from previous production processes, and “leftovers” from previous cutting processes, i.e. trim loss which was sufficiently long enough that it wasn’t considered as waste. Secondly, the actual cutting plan is determined. It depicts which of the available materials actually have to be used and how they have to be cut down.

The finished products are not kept separately in the warehouse, but are stocked in cassettes, together with products of different qualities and sizes. These complete cassettes have to be moved from the warehouse to the cutting area, where the required pieces are taken out of them. Afterwards, the cassettes with the non-required pieces have to be returned to the warehouse. It is very common that the gluelams which have been selected for a customer order have to be taken from different cassettes.

The cutting process itself can be characterized as one-dimensional (cf. Dyckhoff (1990); Wäscher, Haußner, Schumann (2007)). The input to the process, i.e. the finished products from the warehouse (large objects), has to be cut down in one dimension (“length”) only in order to provide the sizes requested by the customers (small items). Output of the process which is different to any size ordered by the customers represents trim loss. If it is of “reasonable” length, it can be stocked in the warehouse and used again as input (then called a “residual piece” or “leftover”) in subsequent cutting processes. This gives an explanation for the large number of different lengths that can be found in the warehouse. Trim loss which is considered to be too short for being used in future represents waste.

The management of Nordlam GmbH regarded the current process of planning and carrying out the cutting process as insufficient. In particular, it was criticized that the cost of planning and the cost of waste were too high. Planning so far was carried out manually each working day, usually three days ahead of the actual process. One of the employees (the “planner”) would decide which of the customer orders was to be cut, which inputs were to be used and which cutting plans were to be applied. For doing so, usually a complete working day was required. The resulting waste of the cutting processes summed up to five percent of the used material, while competitors managed to work with two percent only. This is a significant difference, since – for Nordlam GmbH – one percent of waste represents a value of 45,000 Euros per year. The authors of this paper were therefore asked to analyze the problem and provide a decision support system.

3 Problem Analysis

3.1 Goals of the Cutting Process

Generally, the management requested that all customer orders are to be satisfied exactly, while the corresponding decision-relevant costs related to the respective cutting plans are to be minimized. The first cost category which has to be considered as decision-relevant includes *material costs*. Material costs arise as far as output lengths are provided by the cutting process which represent either a length of a customer order or waste. The costs for providing the customer orders are fixed (since all orders have to be satisfied exactly), thus the decision-relevant material costs consist solely of the cost of (material) waste. They can be considered as proportional to the amount of waste. The cost per length unit of the finished product is known.

The second decision-relevant cost category is made up of *transportation and handling costs*, which are related to the movement of the necessary cassettes from their location in the warehouse to the cutting area and back to the storage. These costs will be referred to as handling costs. They can be assumed as proportional to the number of cassettes which have to be moved. The handling costs per cassette are known.

Residual pieces result in transportation costs, but also in additional (decision-relevant) *inventory costs*. In the first place, inventory costs consist of calculative interests on the respective capital which is tied up in the material. They can be considered as being proportional to the amount of the stocked material and the time for which it is kept in stock. The latter, however, cannot be specified because it depends on the time it takes until an appropriate customer order comes in for which it can be used, which is random. Therefore, as an auxiliary goal, one tries to minimize the amount of material included in newly generated residual pieces.

3.2 Relationships Between Goals

Due to the impossibility of capturing all decision-relevant cost categories in a single objective function, the question arises how the provision (or the reduction) of additional residual pieces (and probably other cost factors) will be affected by a minimization of (the sum of) material and handling costs. In this respect, in particular the effects on the utilization of standard material will have to be analyzed.

Smaller amounts of waste (and, thus, less material costs) can generally be obtained by replacing cutting patterns which would create waste by patterns which would result in a residual length. This would facilitate the input of (long) standard material rather than the input of (short) residual pieces from previous cutting processes. Low handling costs can be achieved by a small number of cassette movements. Again, this will result in facilitating the input of standard material, since usually several pieces of this kind will be found in the same cassette.

However, neither the input of standard material nor the creation of new residual pieces is particularly advisable. Increasing the input of standard material will reduce the corresponding inventory and, likewise, the future options of satisfactory material utilization. Residual pieces result in additional (but not yet quantifiable) inventory costs without guaranteeing, however, that a good material utilization will actually be achieved later.

Therefore, as a consequence for the development of the decision support system, it has been concluded that the input of standard material and the creation of new residual pieces must be controllable.

3.3 Problem Type

The input of the cutting process consists of material of many different lengths. Only the standard material comes in a single length (24 m), of which – as has been described before – just small numbers should actually be used. Also the number of units of a particular length which are ordered by the customers is usually rather small (close to one). According to the typology of Wäscher, Haußner and Schumann (2007), this cutting problem can be characterized as a "Residual Bin-Packing Problem" (RBPP) or a "Residual Cutting Stock Problem" (RCSP). It has to be characterized further as an extension to these problem types because the preceding transport and handling processes have to be explicitly considered, just like the possibility of creating new residual pieces. Cutting problems which permit the provision of residual pieces have been discussed by Scheithauer (1991), Gradisar, Resinovic and Kljajic (1999), Gradisar and Trkman (2005), Trkman and Gradisar (2007), and by Cherri, Arenales and Yanasse (2007). To the best of our knowledge, however, no paper exists in which the described transport and handling processes have been explicitly considered in the generation of cutting plans.

4 Model and Solution Approach

4.1 Fundamentals

For the cutting problem introduced above, a solution approach based on a linear programming model has been developed and implemented. This approach includes two components which require the planner's input. Firstly, he has to specify the maximal acceptable waste for each particular product family. This decision will depend on what he expects to be the shortest order length in the near future. All shorter trim loss pieces are treated as waste. The planner usually has a sound idea of what he will have to expect.

Residual pieces are treated in an analogous way. Generally speaking, only such pieces are accepted of which it can be expected that they can be well used in the near future. This results in a lower and an upper limit on the admissible length of a residual piece. The upper limit is necessary, in particular, to avoid the cutting of small items from long residual pieces or from standard material. Again, the planner has to provide the corresponding information.

From this information concerning the feasible lengths of waste and residual pieces and from the data concerning the customer orders and the available material, a linear programming model is generated, which will be described in the following section 4.2. It has to be noted that the model is solved repeatedly for different fixed values for the maximal number of pieces of standard material which are permitted as input, starting from an upper bound down to zero. Together with additional information about the cost factors (amount of waste, amount of new residual pieces, number of cassettes to be moved), the corresponding cutting plans are presented to the planner, who has to choose the final solution.

4.2 Model

4.2.1 Index Sets, Constants, Parameters, and Variables

Index Sets

I: index set of (small) item types ($I = \{1, \ldots, m\}$);

J: index set of (large) object types;

K: index set of cassette numbers;

P(j): index set of all feasible cutting patterns that can be applied to object type j ($j \in J$);

R: index set of intervals which define feasible residual lengths.

Constants

a_{ijp}: number of times item type i ($i \in I$) appears in cutting pattern p ($p \in P(j)$) for object type j ($j \in J$);

c^{hand}: handling cost per cassette;

c^{waste}: cost of waste per length unit;

c^{resid}: cost of residual pieces per length unit;

c^{trim}_{jp} : cost of trim loss caused by cutting pattern p ($p \in P(j)$) for object type j ($j \in J$);

$$c^{trim}_{jp} = \begin{cases} c^{waste}(L_j - \sum_{i \in I} l_i \cdot a_{ijp}), & \text{if pattern p } (p \in P(j)) \text{ contains a piece that is considered to be waste (cf. (1)),} \\ c^{resid}(L_j - \sum_{i \in I} l_i \cdot a_{ijp}), & \text{if pattern p } (p \in P(j)) \text{ contains a residual piece (cf. (2)),} \\ 0, & \text{else;} \end{cases}$$

d_i: demand for item type i ($i \in I$);

l_i: length of item type i ($i \in I$);

s_{jk}: supply of object type j ($j \in J$) in cassette k ($k \in K$);

t_{jp}: length of trim loss of cutting pattern p ($p \in P(j)$) applied to object type j ($j \in J$); length of object type j not covered by items if pattern p is applied to object type j;

L_j: length of object type j ($j \in J$).

Variables

x_{jp}: number of times cutting pattern p (p $\in$ *P*(j)) is applied to object type j (j $\in$ *J*);

y_k: indicator of utilization of cassette k (k $\in$ *K*);

$$y_k = \begin{cases} 1, & \text{if at least one large object is to be taken from cassette k,} \\ 0, & \text{else.} \end{cases}$$

Parameters

WLMAX: maximal length of trim loss to be accepted as waste;

$ONMAX_j$: maximal number of large objects of type j (j $\in$ *J*) to be accepted in the solution;

$RLMIN_r$: minimal length of trim loss in interval r (r $\in$ *R*) to be accepted as a residual piece;

$RLMAX_r$: maximal length of trim loss in interval r (r $\in$ *R*) to be accepted as a residual piece;

With respect to these parameters, the following conditions must hold:

$$WLMAX < \min(RLMIN_r,\ r \in R),$$

$$RLMIN_r < RLMAX_r,\ r \in R.$$

4.2.2 Cutting Patterns

A *cutting pattern* p (p $\in$ *P*(j)) for object type j (j $\in$ *J*) is a vector

$\mathbf{a_{jp}} := \mathbf{(}a_{1jp}, a_{2jp}, \ldots, a_{mjp}\mathbf{)}$

that satisfies the following properties:

$$\sum_{i \in I} l_i \cdot a_{ijp} + t_{jp} = L_j, \qquad j \in J, p \in P(j);$$

$$\sum_{i \in I} a_{ijp} \geq 1, \qquad j \in J, p \in P(j);$$

$$a_{ijp} \geq 0 \text{ and integer}, \qquad i \in I, j \in J, p \in P(j);$$

$$t_{jp} \geq 0, \qquad j \in J, p \in P(j).$$

A cutting pattern $\mathbf{a_{jp}}$ is called *feasible* if one of the following conditions holds:

- The available object (type) length L_j is used completely, i.e.

$$t_{jp} = 0 \Leftrightarrow \sum_{i \in I} l_i \cdot a_{ijp} = L_j,$$

- the trim loss is "small enough" to be accepted as waste, i.e.

$$0 < t_{jp} = L_j - \sum_{i \in I} l_i \cdot a_{ijp} \le \text{WLMAX}, \tag{1}$$

- the trim loss is of a length that is accepted as a residual piece, i.e.

$$\text{RLMIN}_r \le t_{jp} = L_j - \sum_{i \in I} l_i \cdot a_{ijp} \le \text{RLMAX}_r,\ r \in R. \tag{2}$$

4.2.3 Optimization System

$$\min \sum_{j \in J} \sum_{p \in P(j)} c_{jp}^{trim} \cdot x_{jp} + \sum_{k \in K} c^{hand} \cdot y_k \tag{3}$$

$$\text{s.t.} \quad \sum_{j \in J} \sum_{p \in P(j)} a_{ijp} \cdot x_{jp} = d_i, \quad i \in I; \tag{4}$$

$$\sum_{p \in P(j)} x_{jp} \le \sum_{k \in K} s_{jk} \cdot y_k, \quad j \in J; \tag{5}$$

$$\sum_{p \in P(j)} x_{jp} \le \text{ONMAX}_j, \quad j \in J; \tag{6}$$

$$x_{jp} \ge 0 \text{ and integer}, \quad j \in J, p \in P(j); \tag{7}$$

$$y_k \in \{0,1\}, \quad k \in K. \tag{8}$$

The objective function (3) captures the decision-relevant costs. The first term includes the cost of waste (for "short" pieces of trim loss) and inventory (for the "long" pieces of trim loss which go back into stock as residual pieces; cf. the definition of c_{jp}^{trim}), while the second represents the handling costs for the cassette movements. Constraints (4) are demand constraints which make sure that the customer demands are exactly fulfilled. Each constraint of (5) can be characterized as a supply constraint. It guarantees that for each material type only the available number of pieces is used. Whenever at least one object has to be taken from cassette k ($k \in K$), the corresponding variable y_k will be set to one, and – as a consequence – the respective handling costs will be considered in the calculation of the objective function value. (6) limits the number of pieces to be used of each material type. Of particular interest is the constraint for j = 0, i.e. for the standard material. Its purpose is to control the input of standard material. ONMAX_0 has to be looked upon as a parameter, which is initialized by $\text{ONMAX}_0 = \sum_{k \in K} s_{0k}$, and then successively reduced to zero. Similarly, the planner may also generate cutting plans for different limits concerning the input of other "long" material pieces (i.e. residual lengths from previous cutting processes) if one thinks it may not be advisable or not economical to use all available pieces of this type. Constraints (7) demand that the number of times a cutting pattern is used should be integer, constraints (8) indicate if a particular cassette k ($k \in K$) is being used.

5 Implementation

The described solution approach has been implemented as an individual PC-application under Microsoft Windows. When a planning cycle is started, the necessary, product family-related data is imported directly from the company's database. Among this data are the lengths and the corresponding quantities demanded by the customers (customer demand), as well as the available inventory together with the cassette number in which it is stored. Table 1 depicts a typical data set for a specific product family of October 2007.

Customer Demand	Inventory	Cassette No.
1 x 3330 mm	1 x 3644 mm	961
2 x 9200 mm	1 x 4036 mm	246
1 x 9600 mm	1 x 4468 mm	206
2 x 10100 mm	2 x 4500 mm	986
1 x 11250 mm	1 x 4536 mm	961
3 x 12600 mm	1 x 5120 mm	246
	1 x 5344 mm	595
	2 x 6000 mm	246
	1 x 9652 mm	986
	1 x 10052 mm	986
	1 x 10284 mm	206
	1 x 13744 mm	206
	1 x 15032 mm	206
	1 x 15444 mm	33
	1 x 21060 mm	986
	1 x 24060 mm	206
	1 x 24060 mm	961
	1 x 24060 mm	986
	7 x 24060 mm	33

Table 1: *Data set of the product family "140 x 240 Sicht (Visual)"*

If necessary, the planner may specify further parameters for each product family in addition to this basic data. On the one hand, this may be the respective costs per length unit of waste; on the other hand, one can specify the feasible trim loss intervals. In the implemented decision support system, specific input windows have been designed which can be used for entering the corresponding data. Fig. 1 shows such an input window for the feasible residual lengths (like in Table 1, all data is given in mm). According to this figure, new residual pieces of lengths between 4 and 20 m may be provided. Furthermore, it has been pre-specified that trim loss shorter than 2 m represents waste. In other words: trim loss between 2 and 4 m and of more than 20 m is excluded.

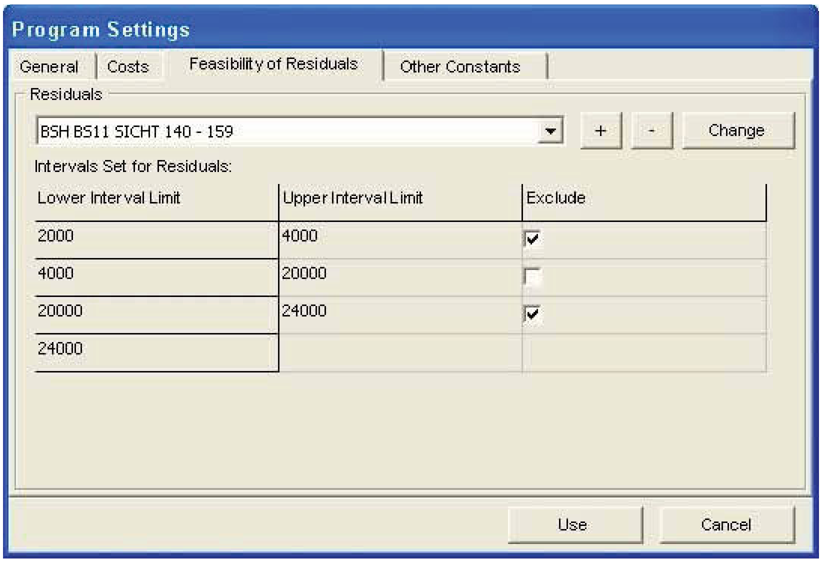

***Fig. 1:** Program Settings for the Residuals*

From the product family-related data, the corresponding optimization model ((3) - (8)) is generated automatically. Optimization runs are performed by means of a commercial LP-software package. The result of the optimization is afterwards presented to the planner (cf. Fig. 2). It is an overview of the solutions (cutting plans) which have been computed for different, previously set values for the maximum number of standard lengths permitted as input. Information on the (sum of) costs of waste and handling on one hand and on the newly provided residual pieces (total length of all new residual pieces in mm) on the other hand exemplifies the (partial) trade-off between the two objectives. Furthermore, for each solution additional characteristic information is given which might be important to the planner, among which is the total quantity of waste (in mm), the necessary number of cassette movements and the number of input pieces needed.

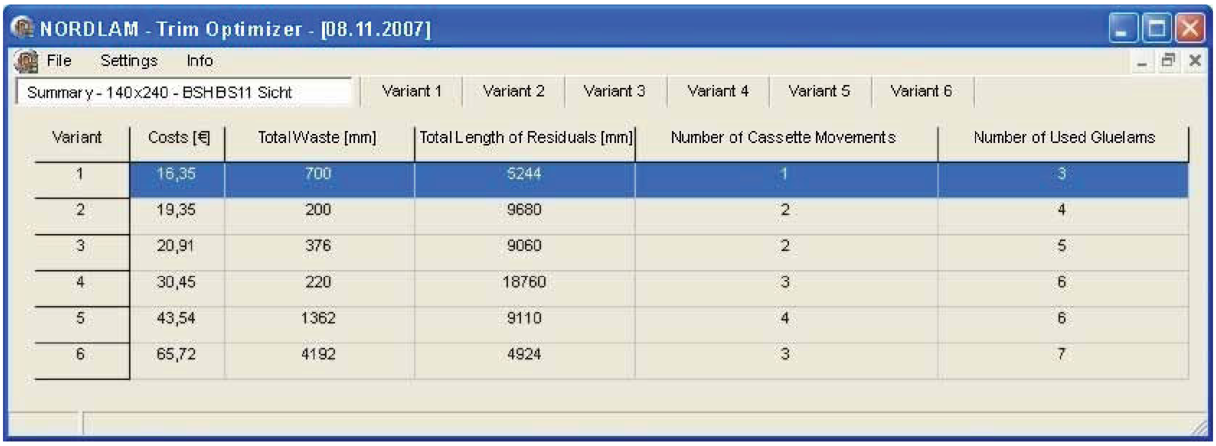

***Fig. 2:** Overview of the solutions*

From the presented solutions the planner has to select one for execution on the shop floor. In order to support him in his decision, the cutting plans related to the different solutions can be displayed. Fig. 3 depicts such a cutting plan, together with the explicit information on waste and handling costs. The waste provided by the plan is also given as a percentage in order to facilitate the comparison with solutions from previous planning cycles.

Fig. 3: *Cutting Plans in Detail*

By pushing the button “execute reservation”, the chosen solution / cutting plan can be finally selected. In the ERP system, the corresponding gluelams will be assigned to the customer orders and the cutting plans are forwarded to the workers in the cutting department.

6 Experience from Practical Utilization and Outlook

The decision support system described above is still in its testing phase. Nevertheless, it is well accepted by the (experienced) planner, which may be attributed to the fact that the system constantly provides cutting plans which are superior to his own, manually-generated plans. Furthermore, solutions are provided in real time. Table 2 demonstrates this fact on an extract from the order data of a particular working day. Each data set is characterized by the total number of pieces ordered and the number of potential input pieces available in stock which may be used for these orders. In order to describe the size of the corresponding optimization model ((3) - (8)), also the number of cutting patterns is given. Finally, the computing times which were needed for the generation of the solution set of Fig. 2 are listed. This includes the times for the generation of the model as well as the time for the optimization runs for all relevant values of the parameter $ONMAX_j$. All computations are performed on a PC with an Intel P4 Processor (3.2 GHz). It is obvious that no significant waiting times arise until the solutions can be presented.

From an economic point of view, the current experience with the decision support system is also very satisfactory. Just by using the “beta version”, the waste could be reduced by one percentage point. The times which will be necessary for the provision of the daily cutting plans are expected to go down from eight to four hours after a full implementation of the system.

Product Family	Number of Ordered Gluelams (Item Types)	Number of Gluelams Available in Stock (Object Types)	Number of Cutting Patterns	Computing Times [sec.]
80 x 200	10	83	28	8,54
80 x 220	6	30	59	3,71
100 x 120	11	65	34	9,09
120 x 120	3	22	289	4,82
120 x 320	6	29	112	2,71
140 x 200	10	35	36	3,36
140 x 240	7	30	153	2,96
160 x 240	11	29	48	4,97
160 x 280	5	26	55	1,26
200 x 240	5	32	113	3,43
200 x 400	5	33	36	1,44
200 x 520	4	25	66	1,59

Table 2: *Example of the Order Data*

Finally, it should not go unmentioned that one data set appeared during the test phase for which the optimization model could not be generated because it contained too many cutting patterns. The existence of such “pathological” data sets is well known to practitioners and researchers in the area of cutting and packing. However, initially this aspect wasn’t considered to be practically relevant for the application and the data structures under discussion. In fact, within six months of utilization of the system, no further data set of this kind has been discovered. Nevertheless, the appearance of this data set demonstrates a potential deficiency of the system. At the moment, such situations would be handled by returning to manual planning. In the future, a column generation approach will be developed and implemented, which refrains from generating all cutting patterns explicitly.

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